Particle Physics II
(LPHY2133)

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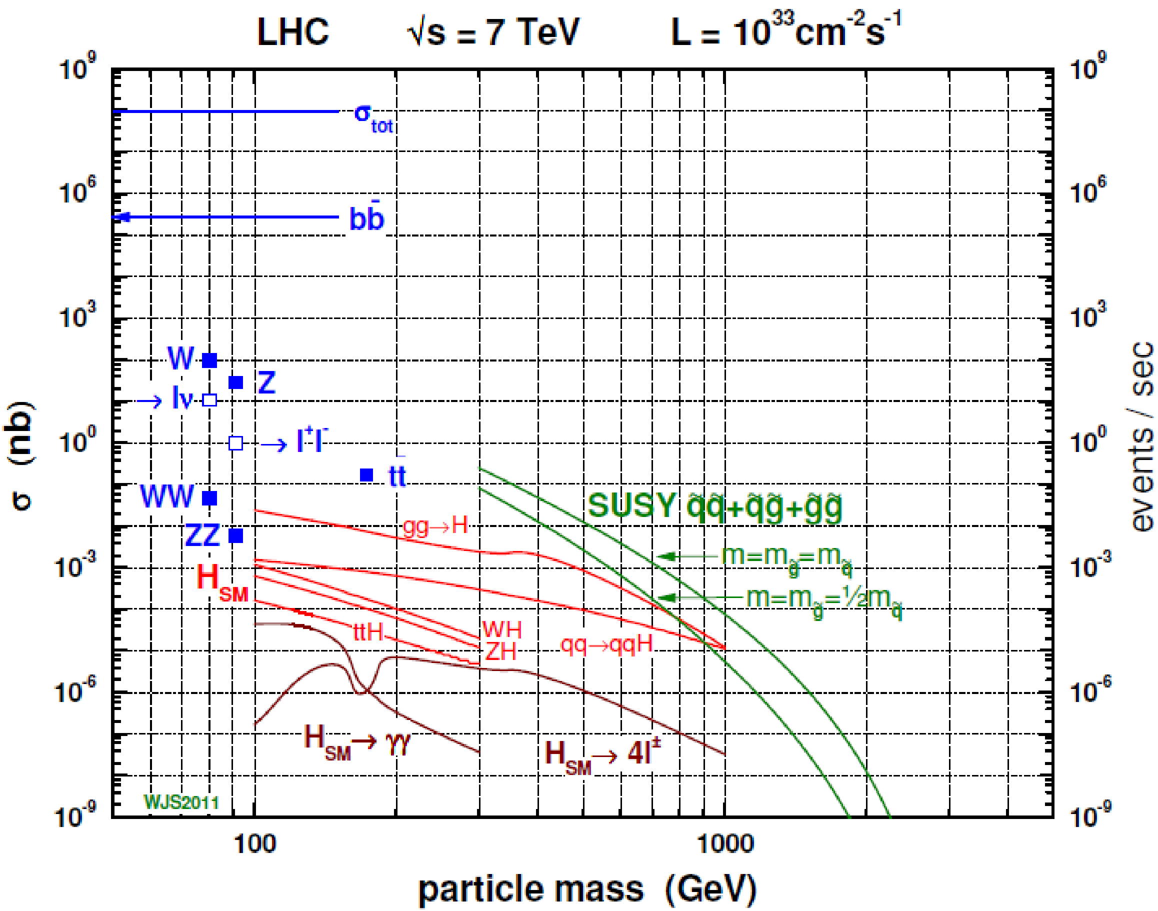
Section 2.5

Dealing with high rates

Triggers



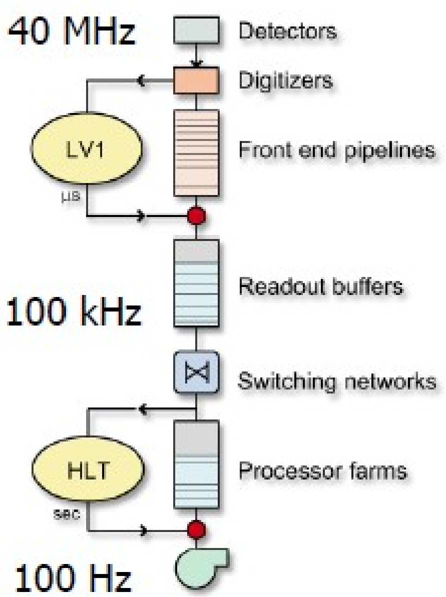
- To generate enough data to ensure Higgs discovery (or exclusion) with Run 1, the LHC has been designed to collide protons every 25 ns ($\Rightarrow$ 40 MHz)
- Multi-purpose detectors have millions of read-out channels $\Rightarrow$ O(1 MB) per bunch crossing $\Rightarrow$ O(100 TB/s)
- No technology is currently able to handle this bandwidth
- Solution: reduce the amount of data online by triggering I/O with fast algorithms based on partial information

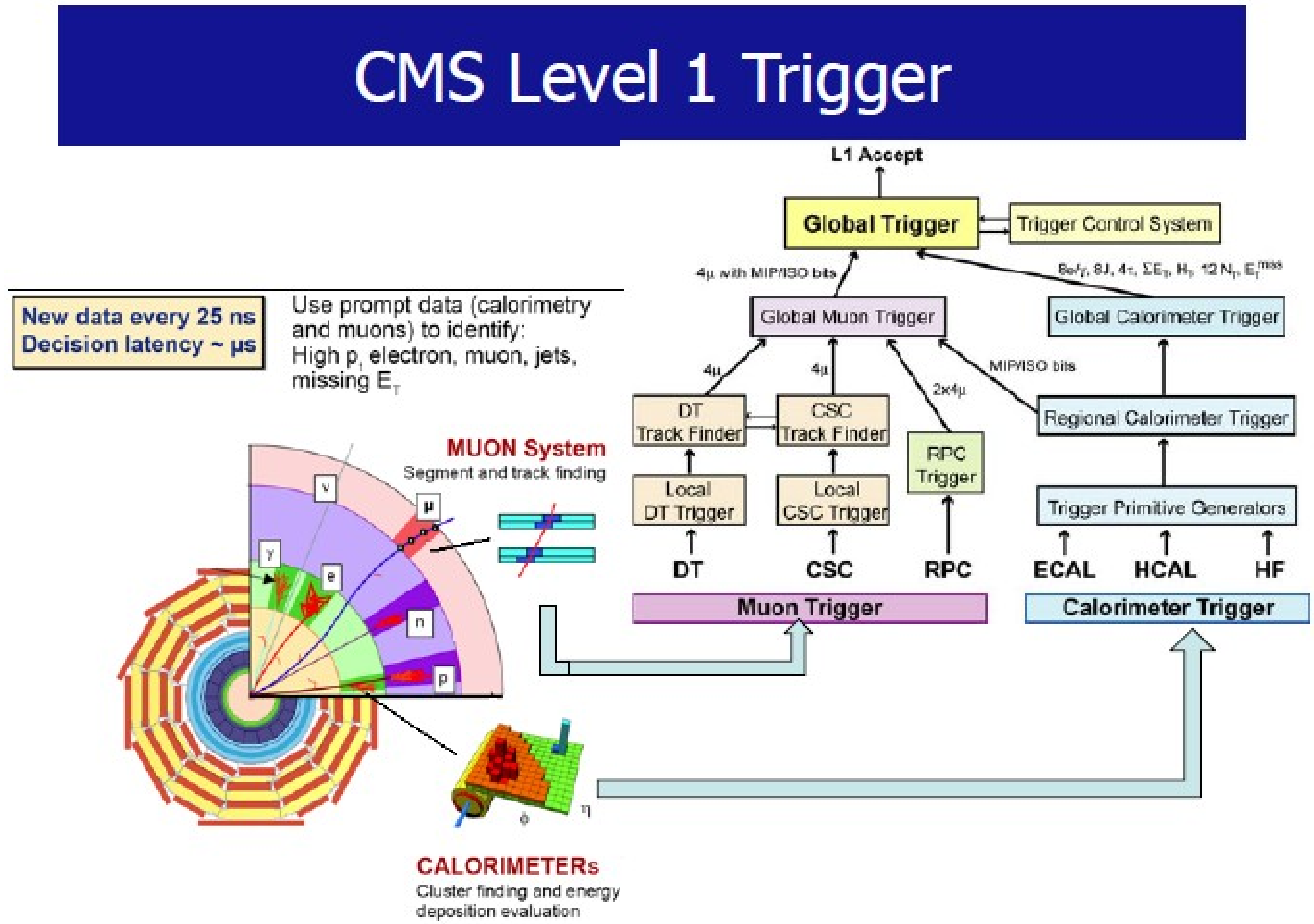
Triggers

- Basic principle: give priority to "rare" events
  - e.g., containing rare final-state particles (muons, electrons, photons, taus, b-jets), or exceptionally energetic hadronic jets, or large momentum imbalance (indirect proof of invisible particles like neutrinos or dark-matter candidates)
- High-rate processes are already well known from previous experiments, so we can ignore them with little regret
  - In reality, we want to save a fraction of them anyway: to test known SM processes at a new energy scale, or to use them as "control data" for cross-checks, or to measure the efficiency of the main triggers in an un-biased sample
  - So we allocate some small bandwidth to "prescaled triggers" (e.g., save 1 event every 10000 that fulfill some conditions)

Trigger

Decision must be **fast**, hence based only on partial information





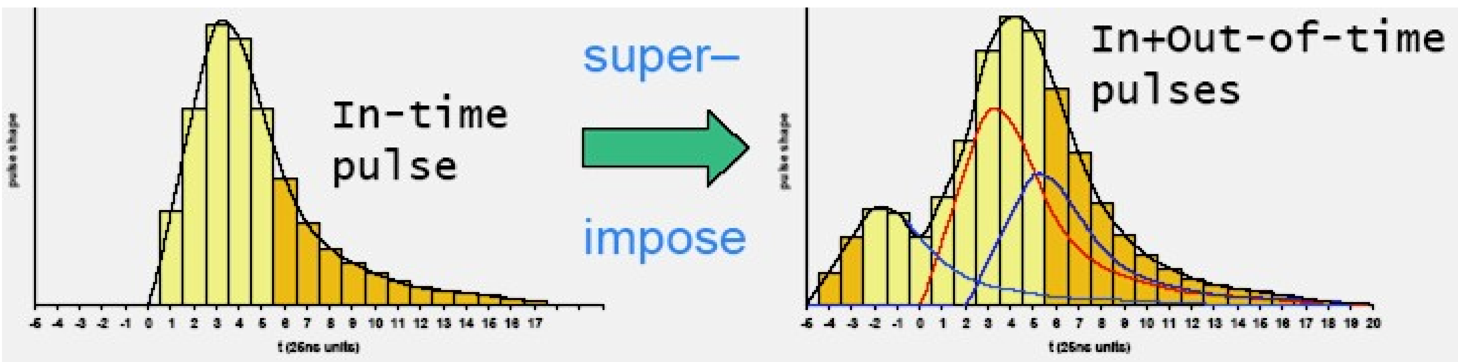
Material from: http://www.hep.ph.ic.ac.uk/~tapper/lecture.html

Trigger usage in analysis

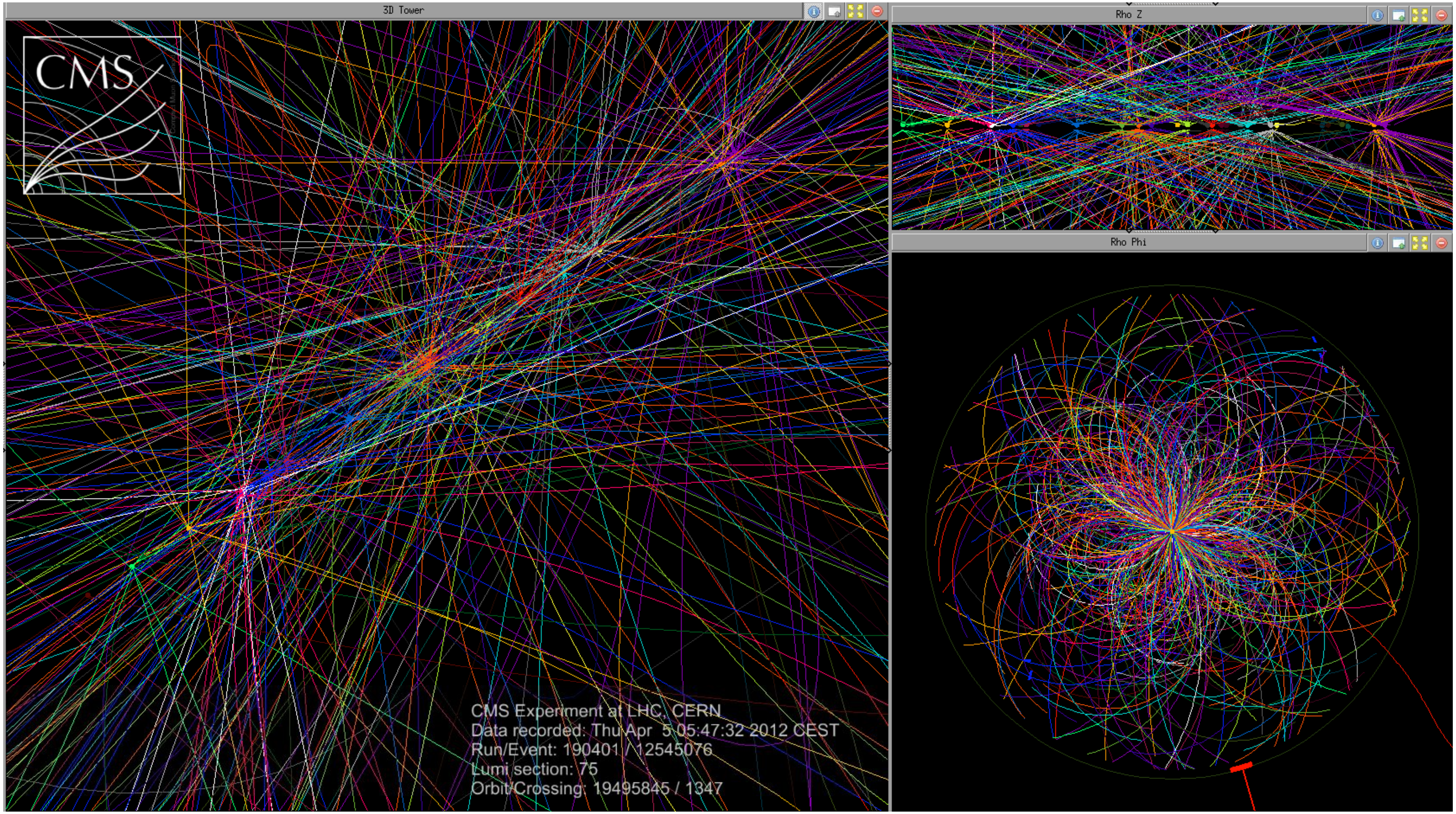
- An event is stored if it passes OR of all trigger algorithms
  - There are O(100) in the multi-purpose LHC experiments
- In practice, the first step of offline selection is the choice of which trigger bits make sense for a given analysis
  - We don't want to spend execution time on events that have no chance to pass the full selection
  - For example, if we need to select 4 leptons, we do not look at triggered events that did not pass any trigger with leptons
  - On the other hand, we usually look at events passing triggers looser than the offline selection, because trigger info is relatively coarse and it may have missed something

A special LHC issue: pile up (PU)

- To achieve large luminosity we need very dense proton bunches (large number of protons, small volume)
- In-time PU: several pp interactions during one bunch crossing
  - We need detectors with precise vertexing
- Out-of-time PU: tail of the electronic signal generated by previous bunch crossings
  - We need detectors with a fast response



Pile up (in-time)



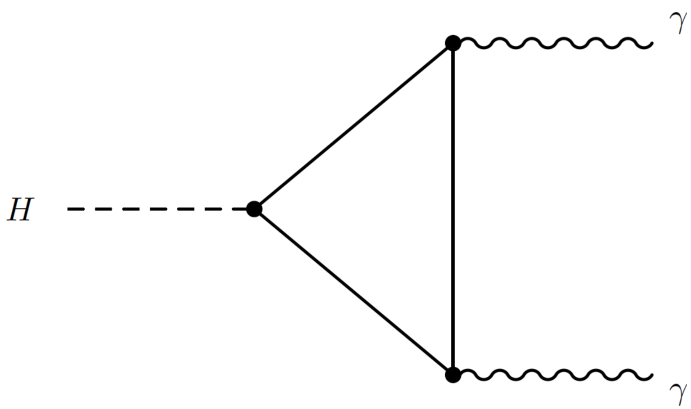
Next topics

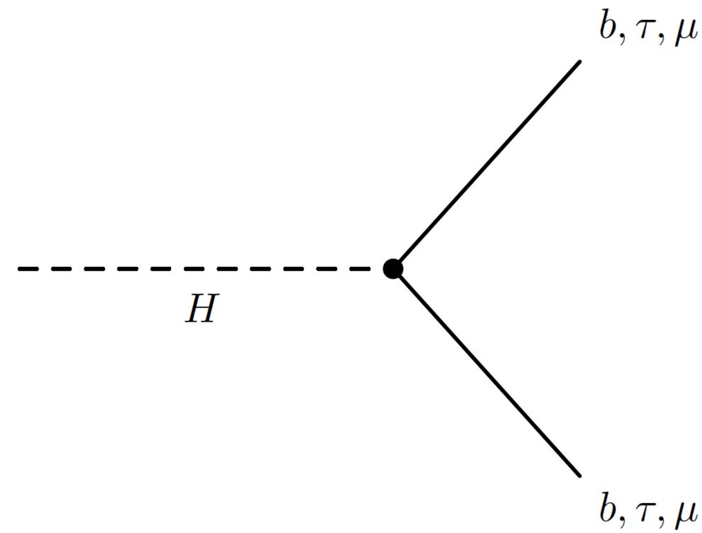
- I will review the most important decay channels of the Higgs boson from the experimental point of view.
- In addition to being the most important from the point of view of statistical significance, they also give complementary information on the Higgs properties
- They are also the occasion to introduce some general detector design concepts
- My pedagogical goal: ideally, after this course you should be able to design an analysis strategy for any given particle, knowing its decays and a range of allowed masses

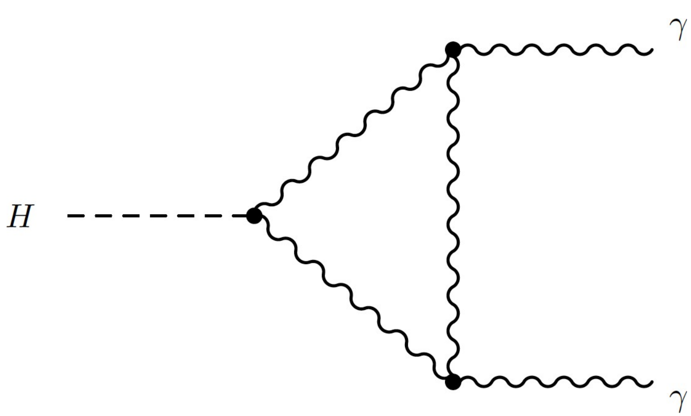
Section 3

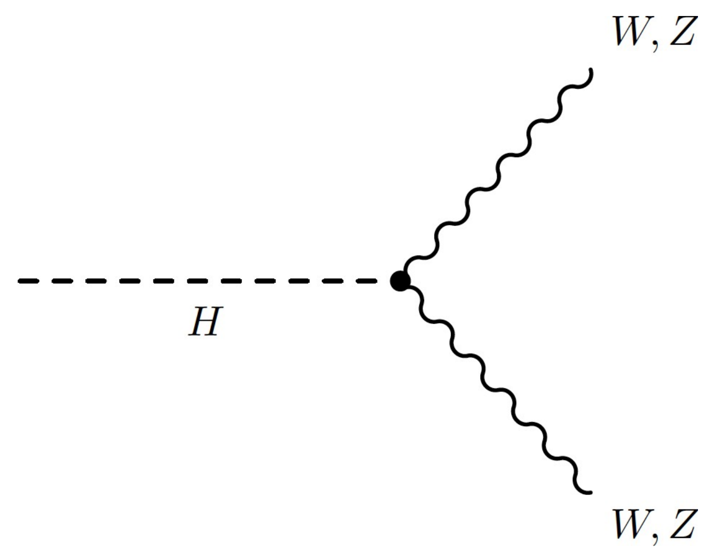
The search for the Higgs boson in the $l^+l^-l^+l^-$ channels ($l = \mu, e$)

Higgs decays



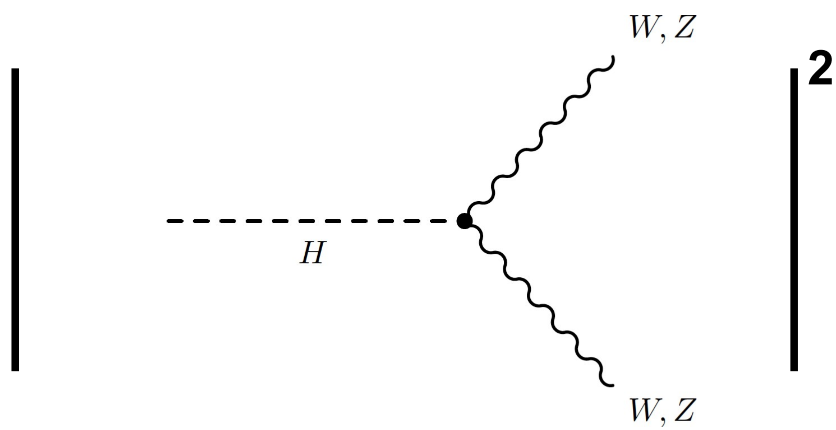






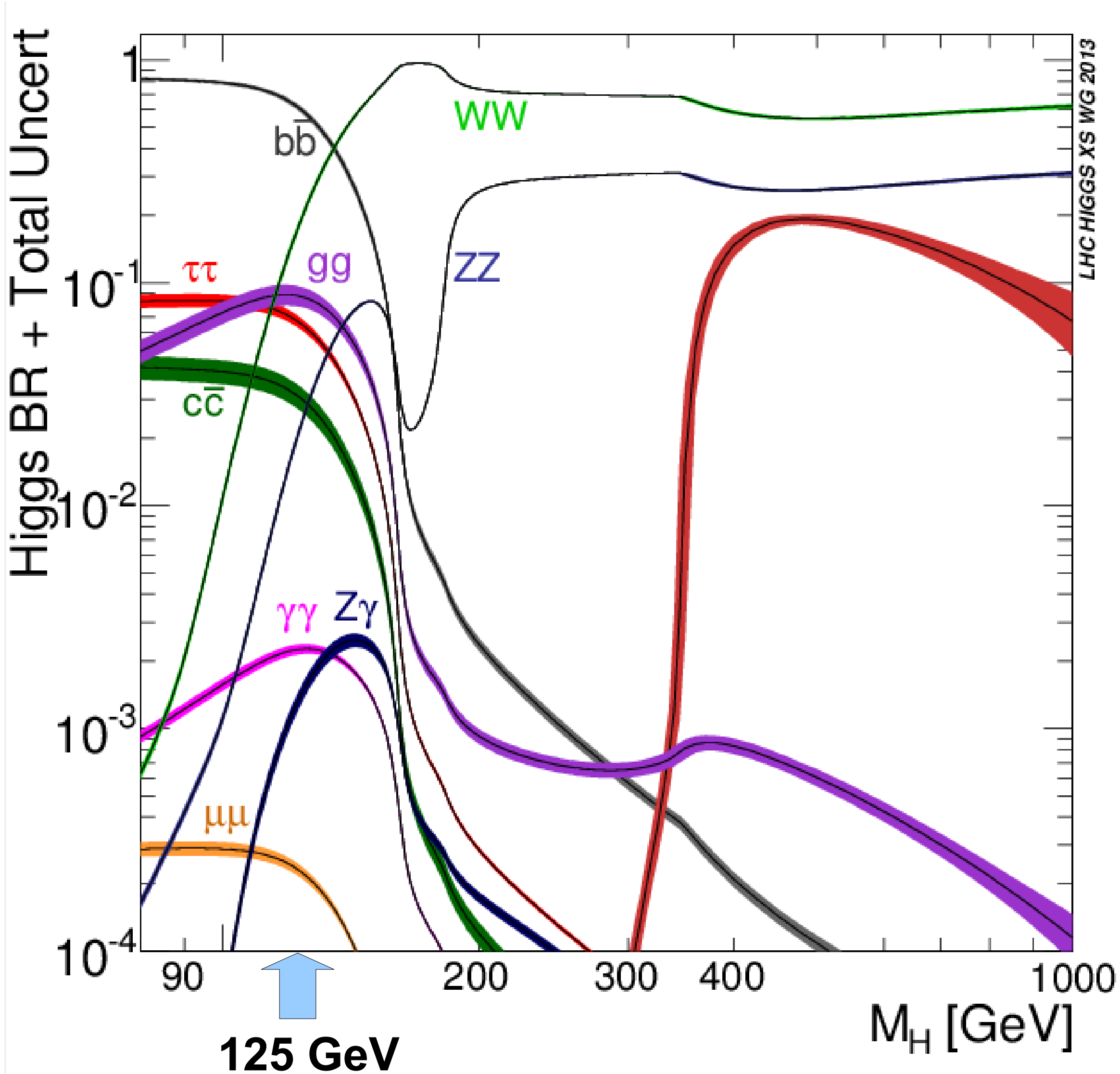
Higgs decay into W, Z

$\Gamma(H\rightarrow VV)\propto$

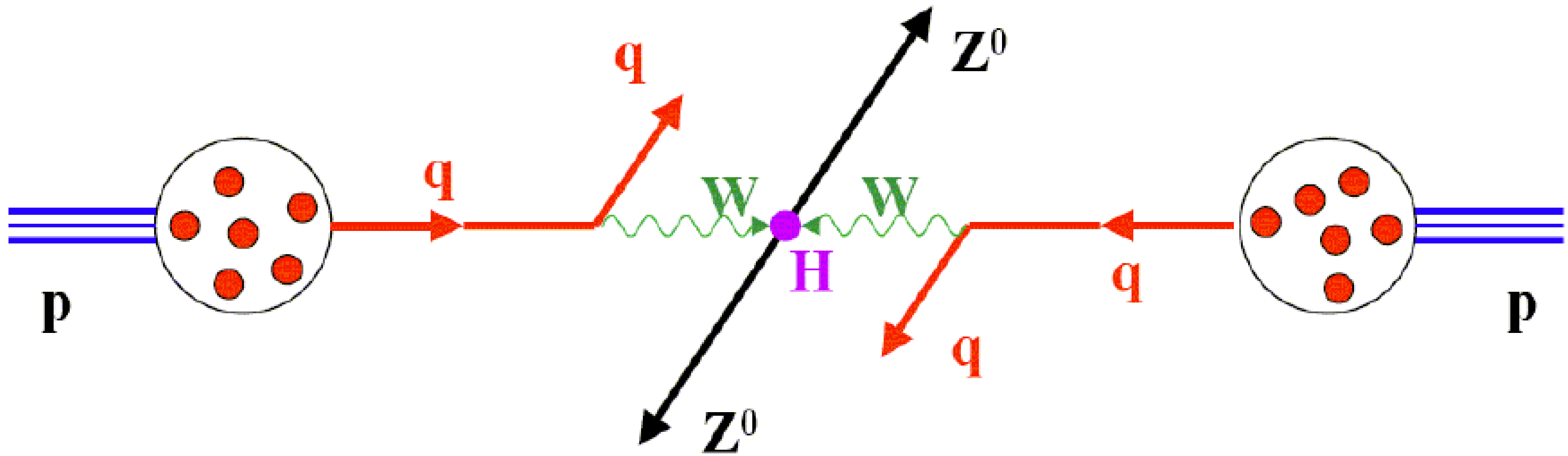


For $m_V < m_H < 2m_V$ one of the two V is real (on-shell) and the other is virtual (off-shell)

Branching ratios vs mass



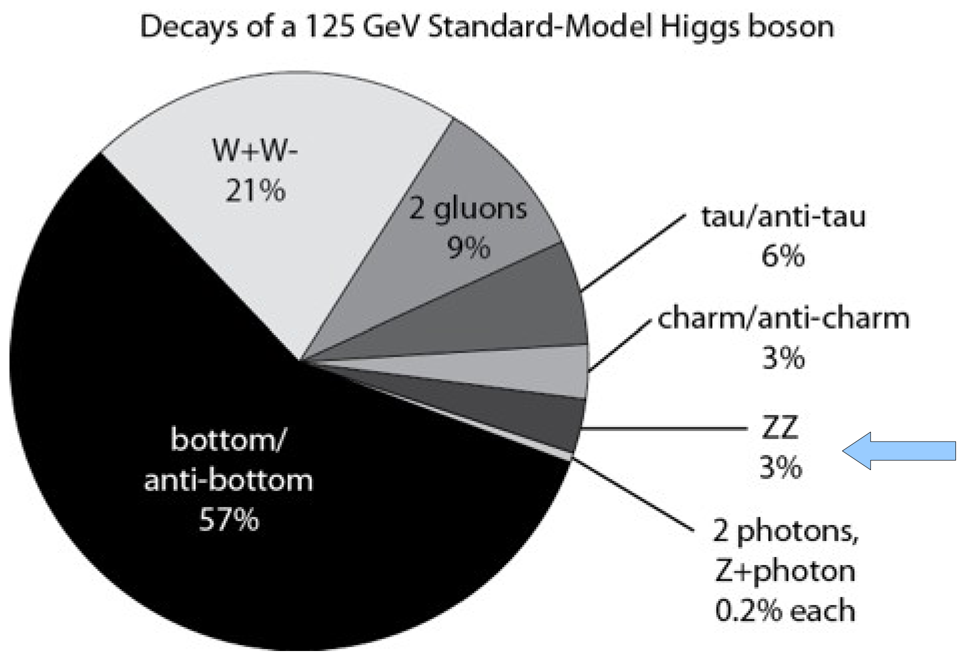
Accelerator and detector choices



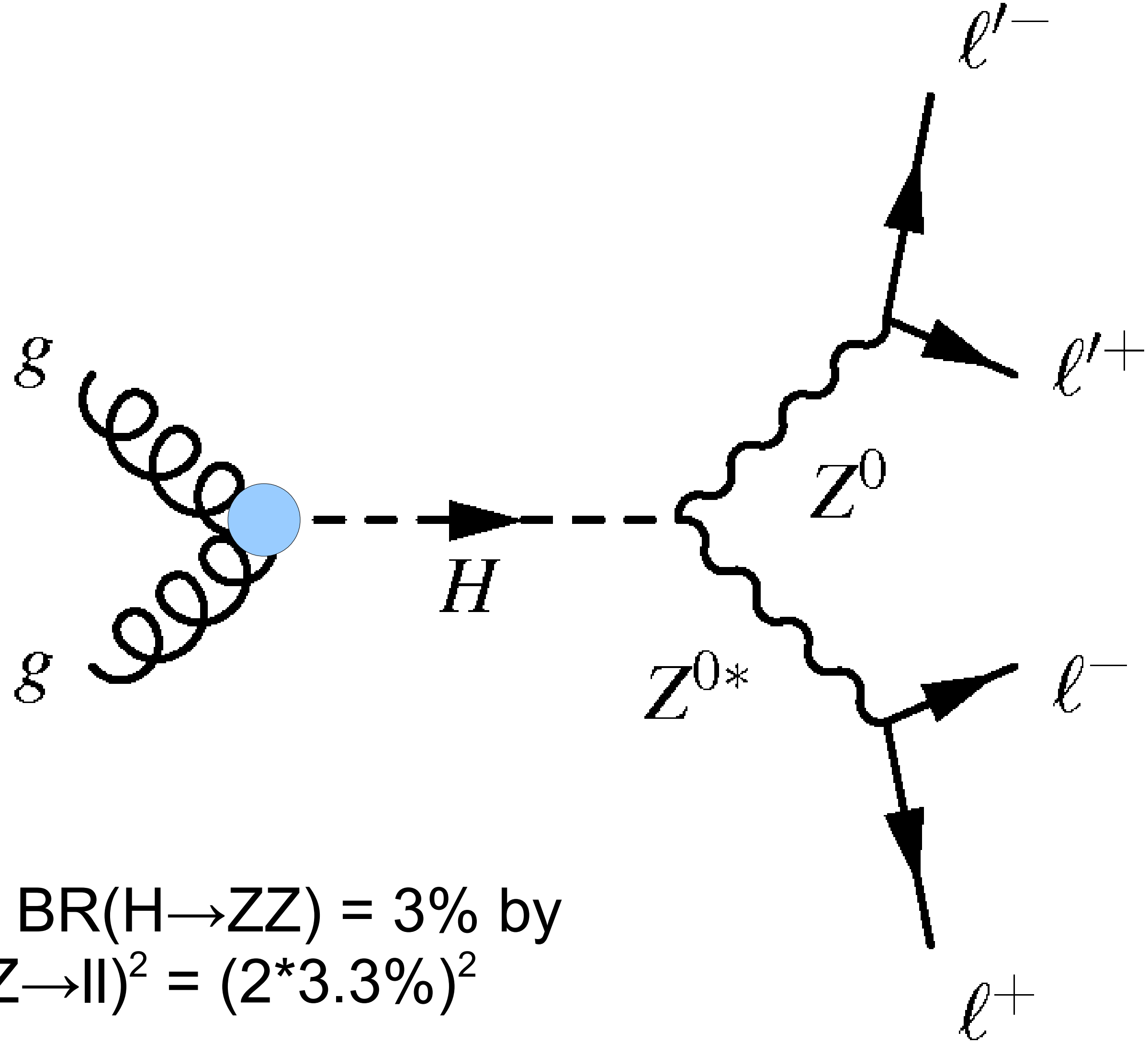
Ensure sensitivity up to M_H ~ 1 TeV (approximate unitarity bound):

- Detectors must be sensitive to Higgs decays up to ~ 500 GeV W and Z decays up to ~250 GeV precise momentum measurement up to that scale detector with large magnetic field and large radius
- Large probability of finding a parton, in the proton, able to radiate a particle (e.g., a W) of ~ 500 GeV parton momentum of O(1 TeV) the proton beams must have multi-TeV energy

Branching ratios @ 125 GeV

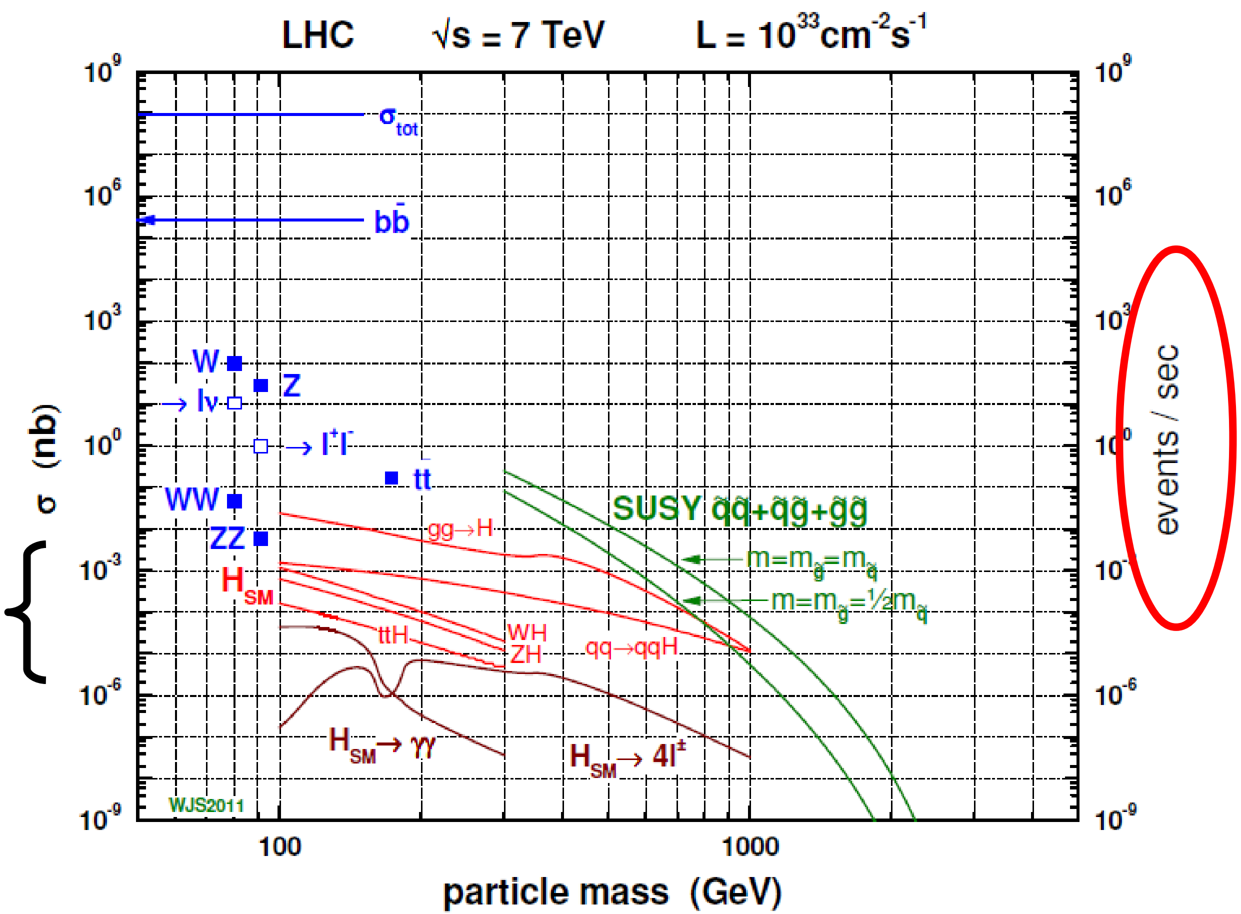


Signal diagram



Multiply BR(H→ZZ) = 3% by
BR(Z→ll)^2 = (2*3.3%)^2

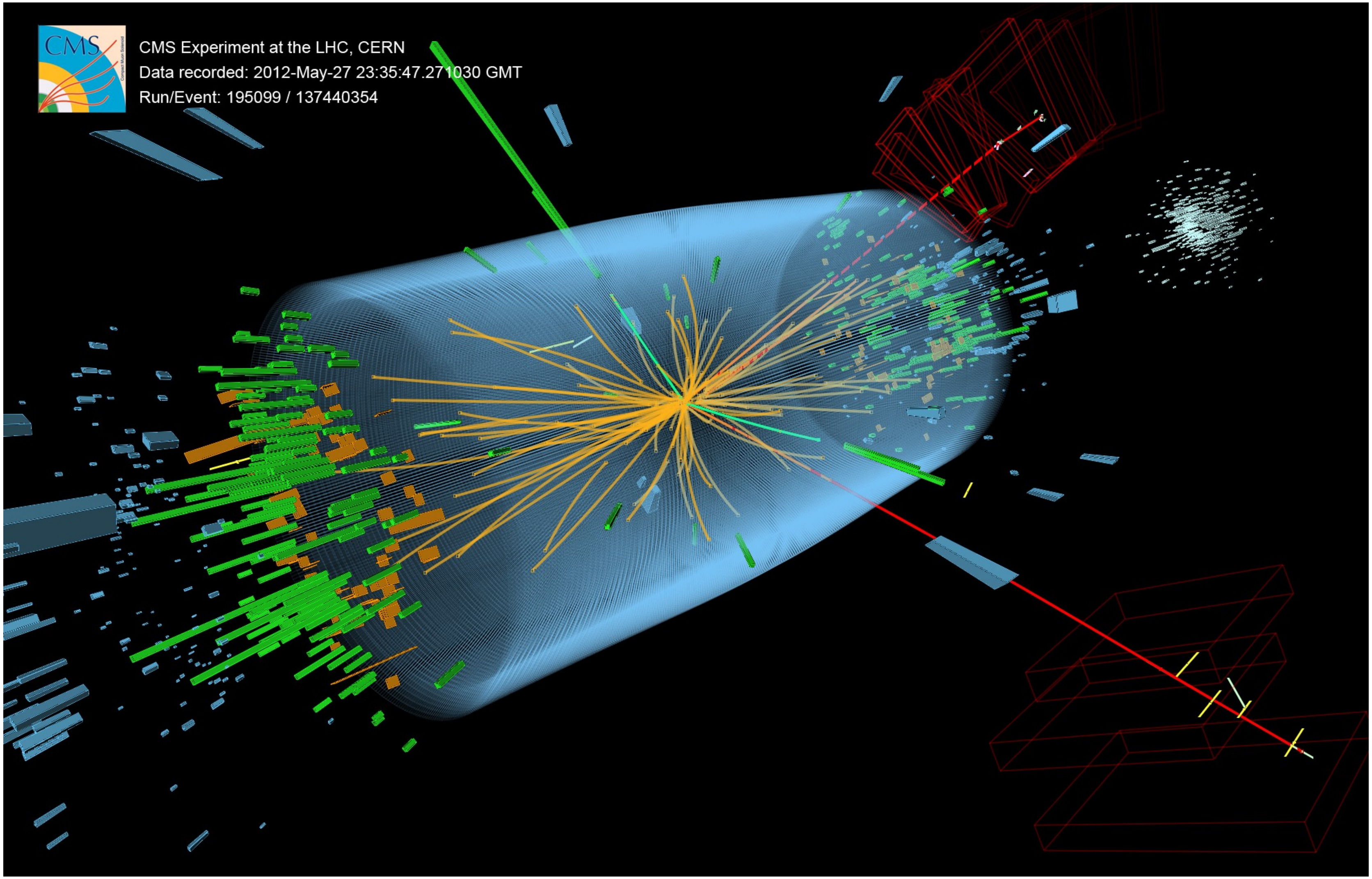
Backgrounds



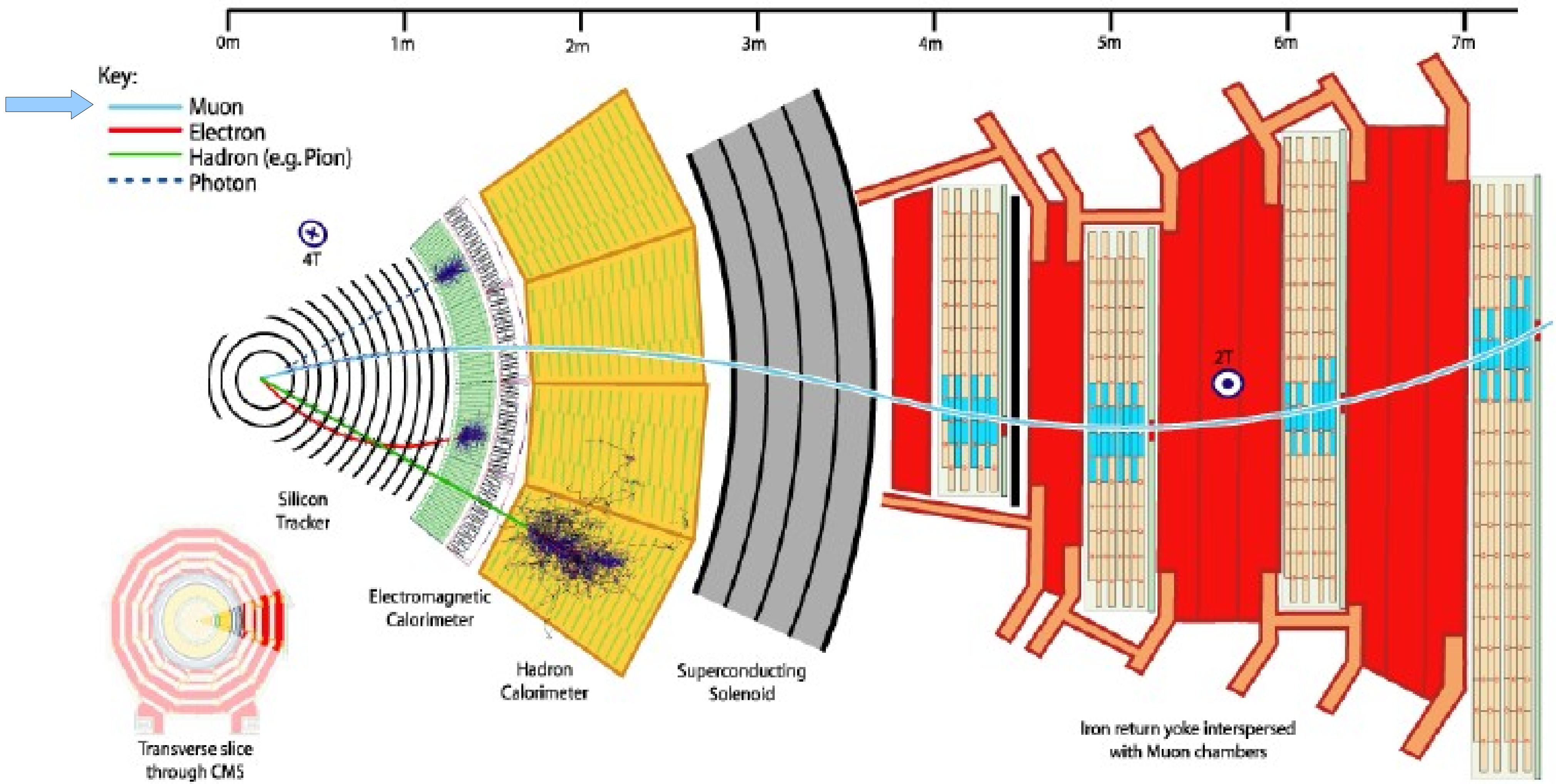
H→ZZ→4l is rare @125 GeV, but continuum ZZ is rare too

This is an old plot; rates are much higher now: larger luminosity (10^{34} $cm^{-2}s^{-1}$); larger energy (13 TeV) → more particles; larger pile-up (i.e., simultaneous pp collisions)

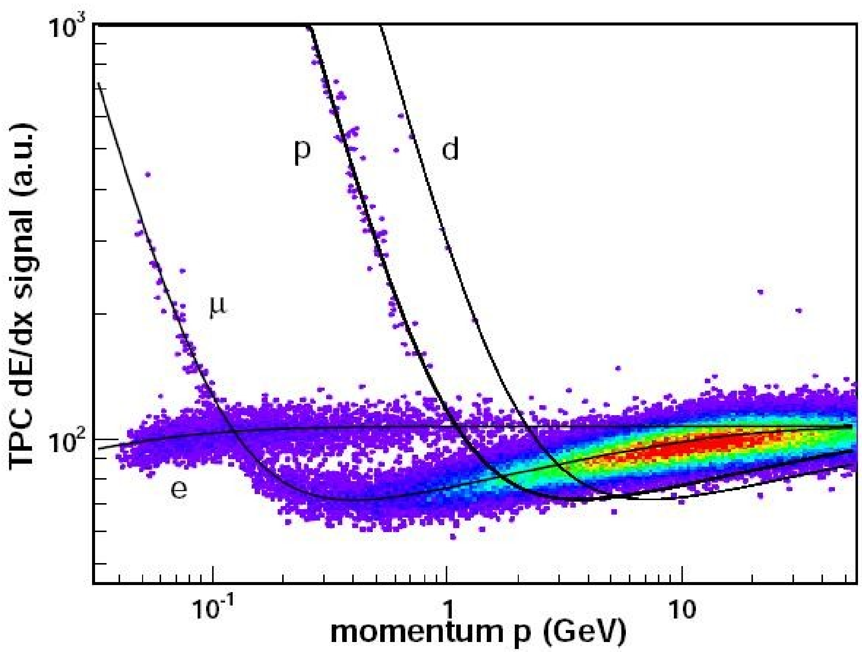
A candidate event

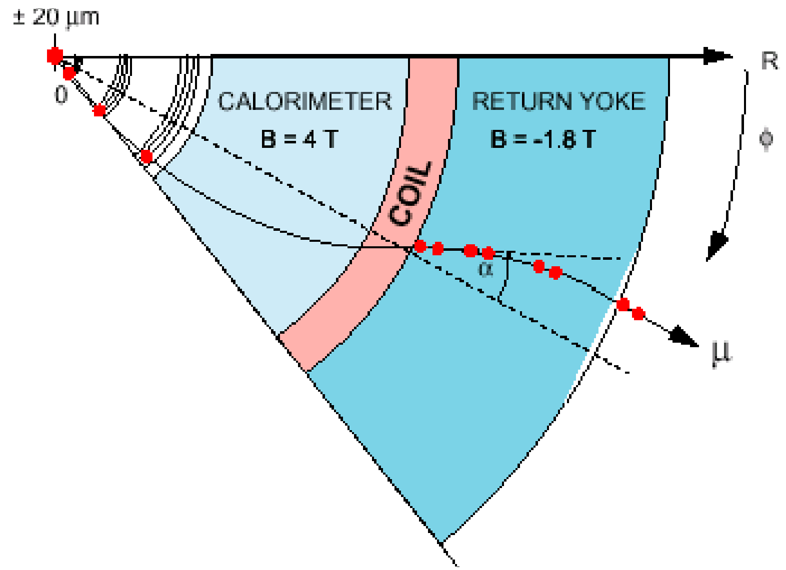


Muon identification



Muon identification



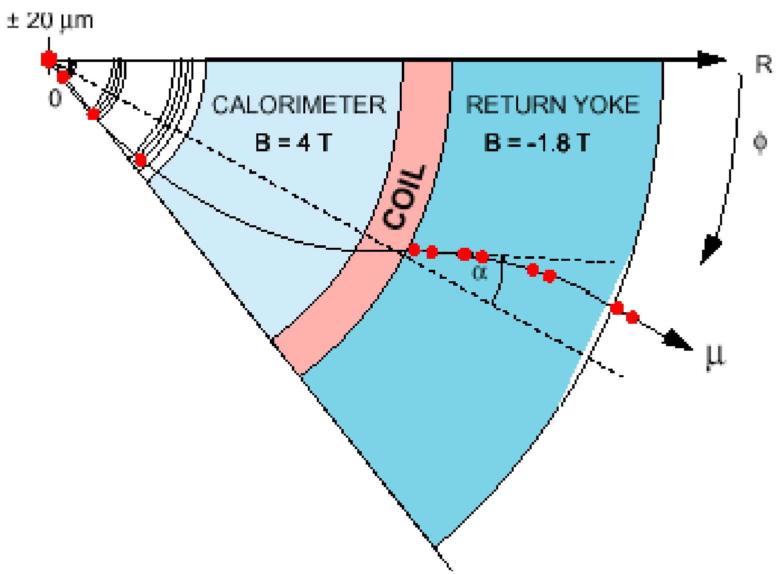


Basic concepts for muon identification:

- A muon does not feel the strong nuclear force
- Its mass gives (by chance) the smallest rate of EM energy loss among all the long-lived particles that we know

⇒ if a particle is seen both before and after passing through a lot of material, there is high probability that it is a muon

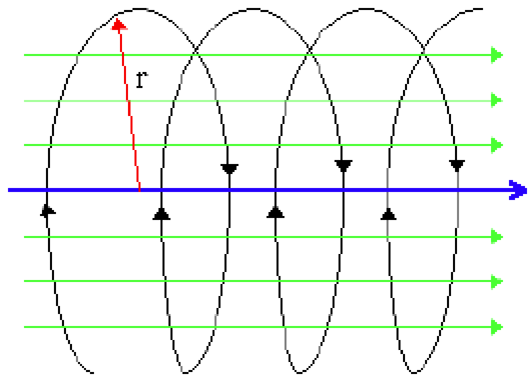
Muon momentum measurement

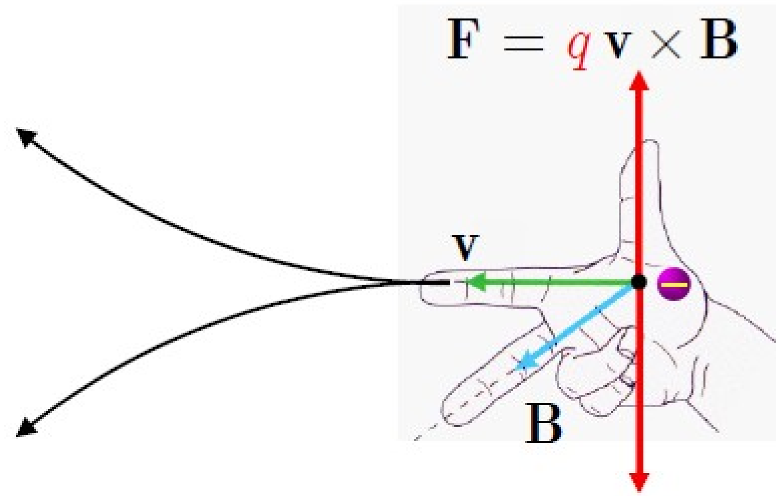


As usual for charged particles, we infer the momentum from the curvature of the trajectory in a magnetic field (principle of the magnetic spectrometer)

As illustrated in this figure for the example of CMS (in ATLAS it is not too different), in the case of a muon we can use two measurements: before and after passing through the calorimeters

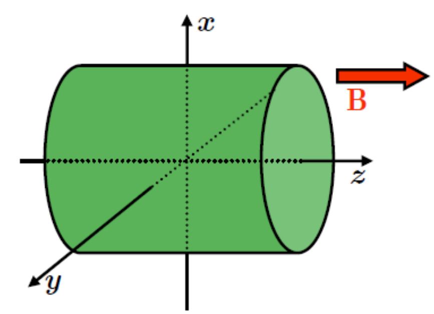
Charged-particle momentum measurement





Basic formula: $q \cdot p_T = B \cdot r$, which becomes $p_T = 0.3 \cdot B \cdot r$ when p_T is in GeV/c, B is in Tesla, R in m, and q=e.

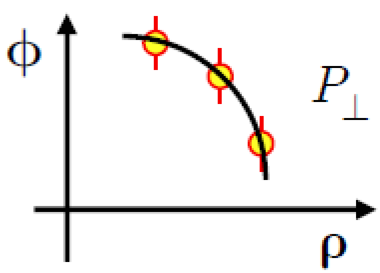
Solenoids create spatially uniform B fields. Both ATLAS and CMS chose this geometry for their *inner trackers*



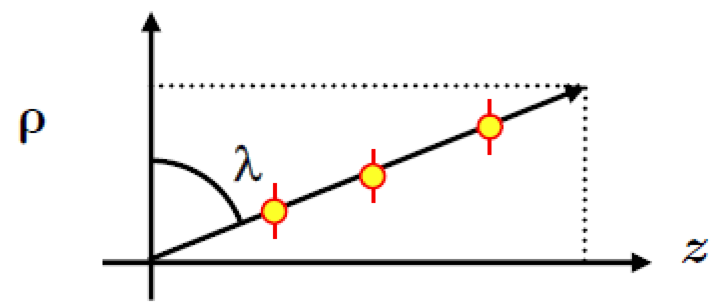
Figures from http://www.physics.iitm.ac.in/~sercehep2013/track2_Gagan_Mohanty.pdf

Charged-particle momentum measurement

Solenoidal field along z: deflection in x-y (or ρ–ϕ) plane



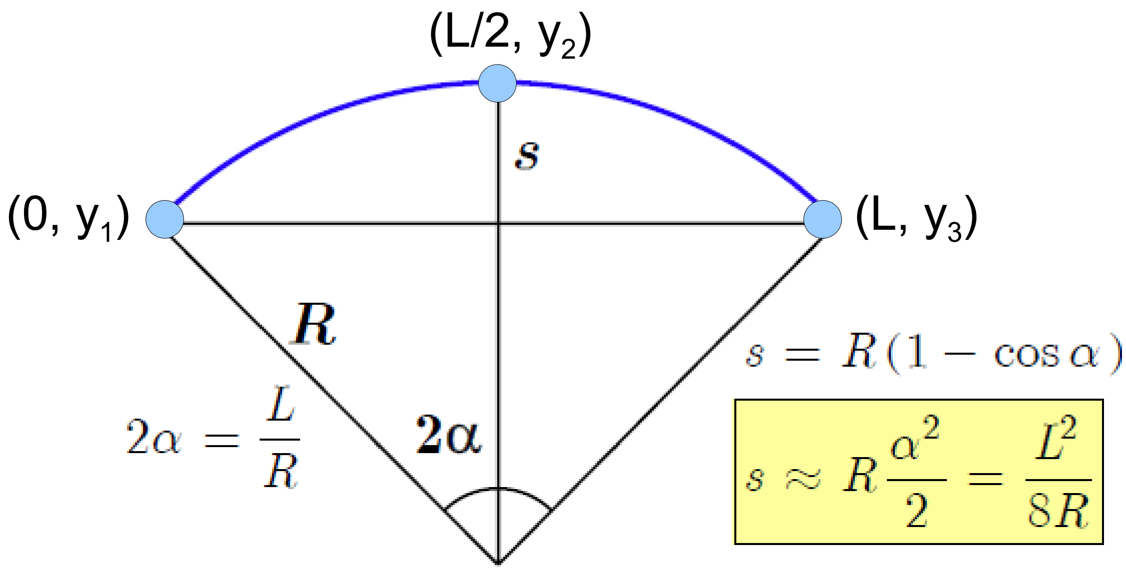
$$P_{\perp} = P\cos\lambda = 0.3BR$$

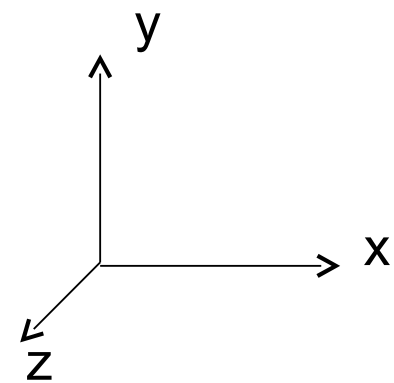


To measure a curvature you need at least 3 points. In the following, to simplify the formulas, I will make an example with just 3 measurements of the trajectory, all having the same position uncertainty.

Charged-particle momentum measurement

Example with just 3 detector layers located at x = 0, L/2, L:





$$s = y_2 - \frac{y_1 + y_3}{2} \approx \frac{L^2}{8r} = \frac{L^2}{8p_\perp/(0.3B)} = \frac{0.3BL^2}{8p_\perp}$$

Momentum uncertainty

$$s = y_2 - \frac{y_1 + y_3}{2} \approx \frac{L^2}{8r} = \frac{L^2}{8p_\perp/(0.3B)} = \frac{0.3BL^2}{8p_\perp}$$

$$\sigma_s = \sqrt{3/2}\,\sigma_y$$

$$\frac{\sigma_{p_\perp}}{p_\perp} = \frac{\sigma_s}{s} = \frac{\sqrt{3/2}\,\sigma_y}{(0.3L^2B)/(8p_\perp)} = \frac{8p_\perp\sqrt{3/2}\,\sigma_y}{0.3L^2B} = 32.6\frac{p_\perp\sigma_y}{L^2B} \ \textbf{(m, GeV/c, T)}$$

Momentum uncertainty

$$\frac{\sigma_{p_\perp}}{p_\perp} = \frac{\sigma_s}{s} = \frac{\sqrt{3/2}\,\sigma_y}{(0.3L^2B)/(8p_\perp)} = \frac{8p_\perp\sqrt{3/2}\,\sigma_y}{0.3L^2B} = 32.6\frac{p_\perp\sigma_y}{L^2B} \quad \textbf{(m, GeV/c, T)}$$

The relative momentum uncertainty grows with momentum itself.
To have excellent momentum precision one needs:

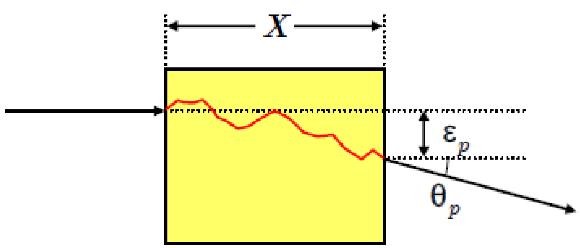
- Large detector size (L)
- Large magnetic field (B)
- Excellent position resolution of the tracking detector (σ_y)

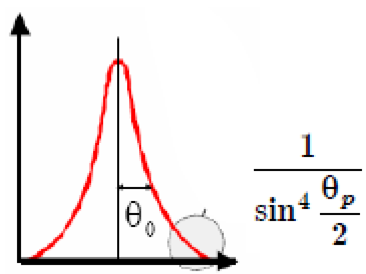
The above expression was derived for n=3 measurements; approximate formula for n>>3 equally spaced points with the same position resolution:

$$\frac{\sigma_{p_\perp}}{p_\perp} = \sqrt{\frac{720}{n+4}}\frac{\sigma_y p_\perp}{(0.3BL^2)} \quad \textbf{(m, GeV/c, T)}$$

Multiple scattering

The previous formulas only consider the component of the momentum uncertainty that comes from the position measurement.
The passage of the particle through the detector also causes an intrinsic uncertainty due to multiple EM deflections by the nuclei of the material.
This is particularly important for muons: remember that they are identified by surviving the passage through a lot of dense material.





$$P(\theta_p) = \frac{1}{\sqrt{2\pi\langle\theta_p^2\rangle}} \exp\left[-\frac{1}{2\langle\theta_p^2\rangle}\theta_p^2\right]$$

Deflection distribution follows Rutherford's law in the tails (single hard scattering) and is ~ Gaussian in the bulk (*Q: why?*)

$$\langle\theta_p^2\rangle = K\frac{X}{X_0}$$

$$K = z^2\left(\frac{0.0136}{p\beta}\right)^2$$

Momentum resolution

Additional uncertainty on the sagitta due to multiple scattering is proportional to about $L\delta\theta$. The complete formula, where L_r is the EM radiation length in the material:

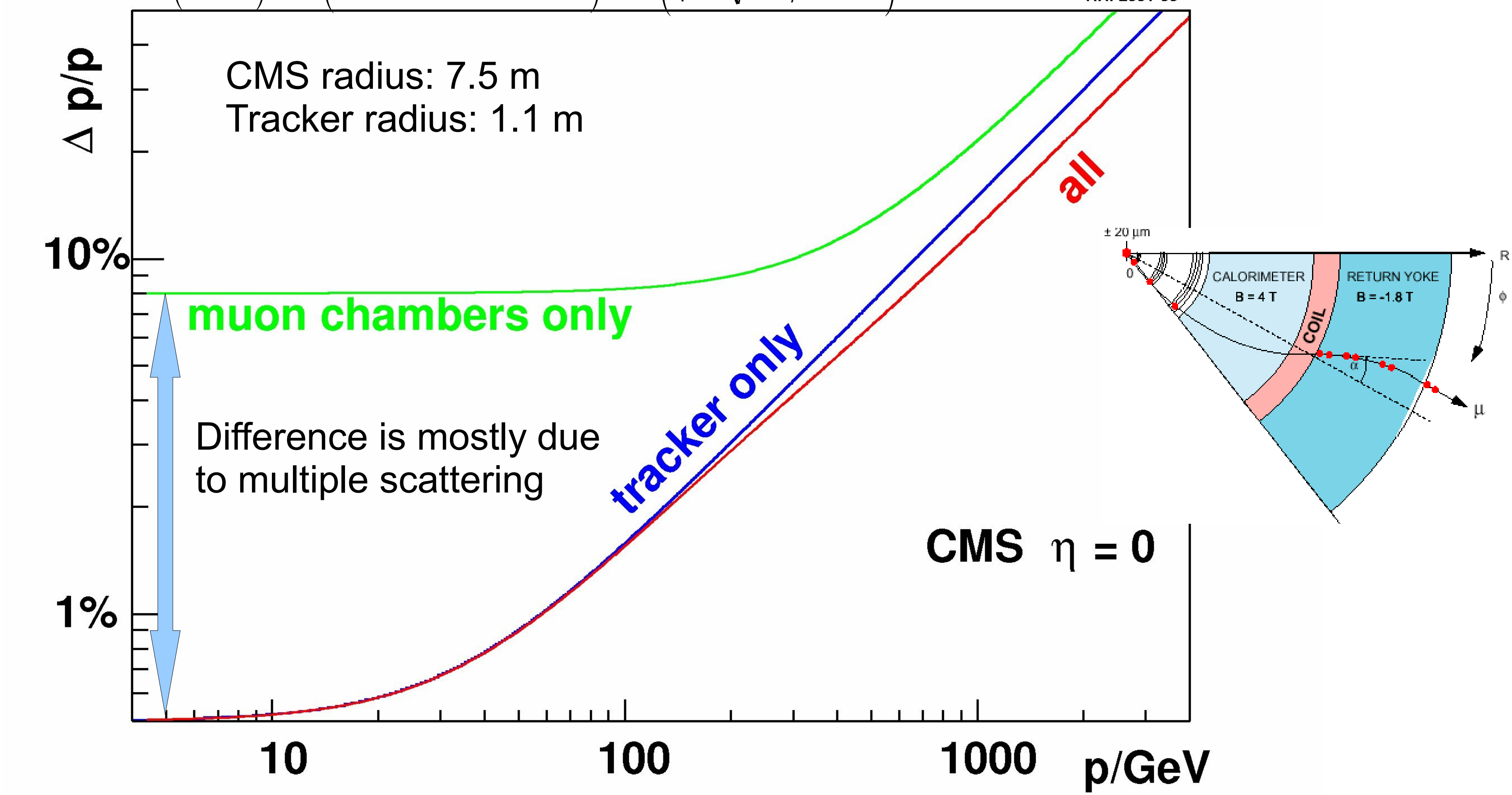
$$\frac{\sigma_{p_T}}{p_T} = \frac{s^{rms}_{plane}}{s_B} = \frac{\frac{L'}{4\sqrt{3}} \frac{13.6 \times 10^{-3}}{p\beta} z\sqrt{L'/L_r}}{0.3BL^2 z/(8p_\perp)} \quad \text{with } L' = L/\sin\theta \text{ , } p_\perp = p\sin\theta$$

$$\left(\frac{\sigma_{p_\perp}}{p_\perp}\right)^2 = \left(\sqrt{\frac{720}{n+4}} \frac{\sigma_y p_\perp}{(0.3BL^2)}\right)^2 + \left(\frac{52.3 \times 10^{-3}}{\beta B\sqrt{LL_r \sin\theta}}\right)^2 \text{(m, GeV/c, T)}$$

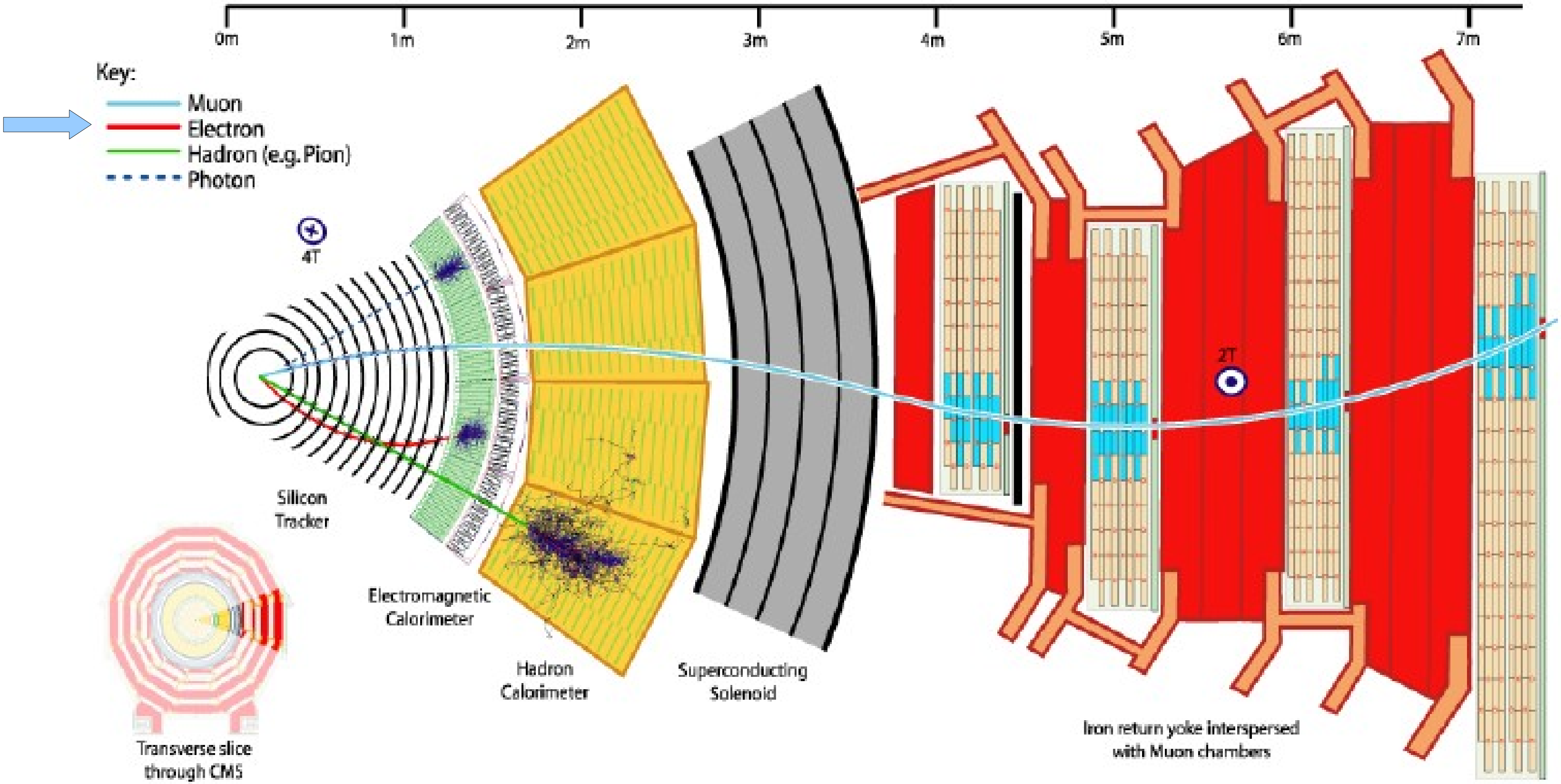
For relativistic particles ($\beta \sim 1$) it is insensitive to momentum. It depends on the material of the detector via L_r

CMS: muon p_T resolution vs p_T

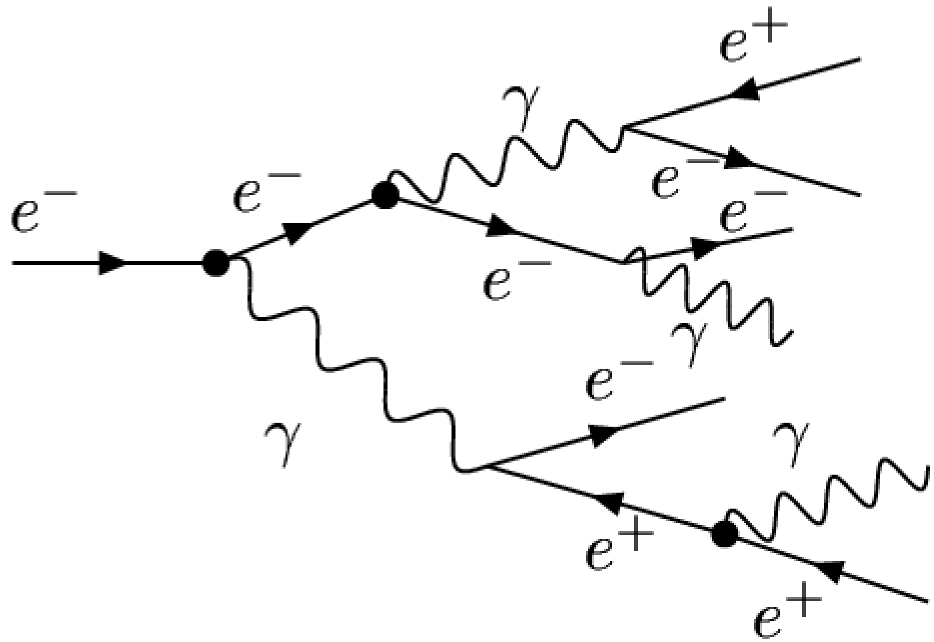
$$\left(\frac{\sigma_{p_\perp}}{p_\perp}\right)^2 = \left(\sqrt{\frac{720}{n+4}}\frac{\sigma_y p_\perp}{(0.3BL^2)}\right)^2 + \left(\frac{52.3\times 10^{-3}}{\beta B\sqrt{LL_r}\sin\theta}\right)^2 \quad \text{(m, GeV/c, T)}$$



Electron identification

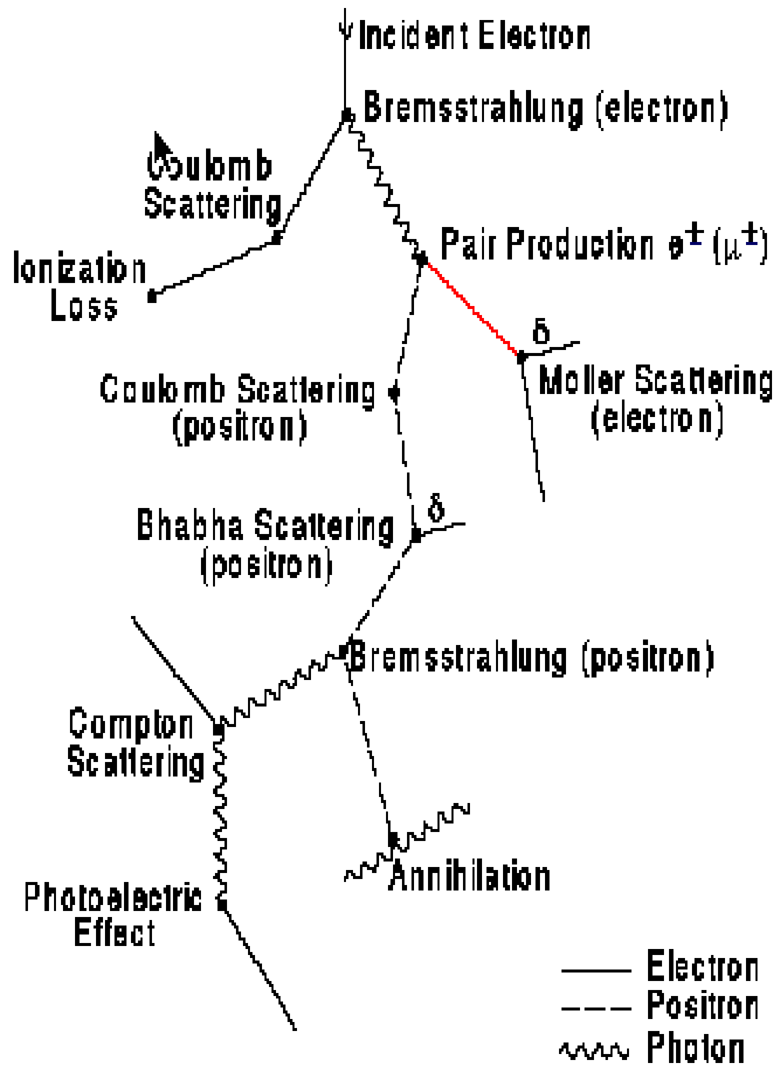


EM shower



Basic principle of electron identification: lightest charged particle
$\Rightarrow$ largest probability to emit photons by bremsstrahlung

EM shower



Incident Electron
Bremsstrahlung (electron)
Coulomb Scattering
Ionization Loss
Pair Production e± (μ±)
δ
Moller Scattering (electron)
Coulomb Scattering (positron)
δ
Bhabha Scattering (positron)
Bremsstrahlung (positron)
Compton Scattering
Annihilation
Photoelectric Effect
Electron
Positron
Photon

Calorimeters

- Basic concept: the particle is stopped; its initial energy is split between several particles in a shower, fully absorbed by the material and transformed in something detectable
- Classification by target particles:
  - *EM cal.*: high-Z material, in order to maximize interaction probability for electrons and photons
  - *Hadronic cal.*: large nuclear cross section for $\pi^{\pm}$,$K^{\pm}$/K_L,p,n, ...
- Classification by detector structure:
  - *Sampling calorimeters*: alternance of passive (absorber) and active (signal formation) layers
  - *Homogeneous calorimeters*: same medium stops the particle and yields a signal proportional to initial energy

Energy uncertainty in calorimeters

$$\frac{\sigma}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c$$

- *Stochastic term* (a): this uncertainty comes from the statistical fluctuations on the number of particles in the shower that pass the threshold to leave a visible signal (n_{vis}); roughly Poisson statistics ($\sigma \propto \sqrt{n_{vis}}$); and $n_{vis} \propto E$
- *Noise term* (b): uncertainties that are independent of incoming particle energy, like detector noise and pile-up
- *Constant term* (c): local non-uniformities in detector response give random effects proportional to E; need to inter-calibrate the calorimeter cells (test beams, then cosmics, then collision data)

Energy uncertainty in calorimeters

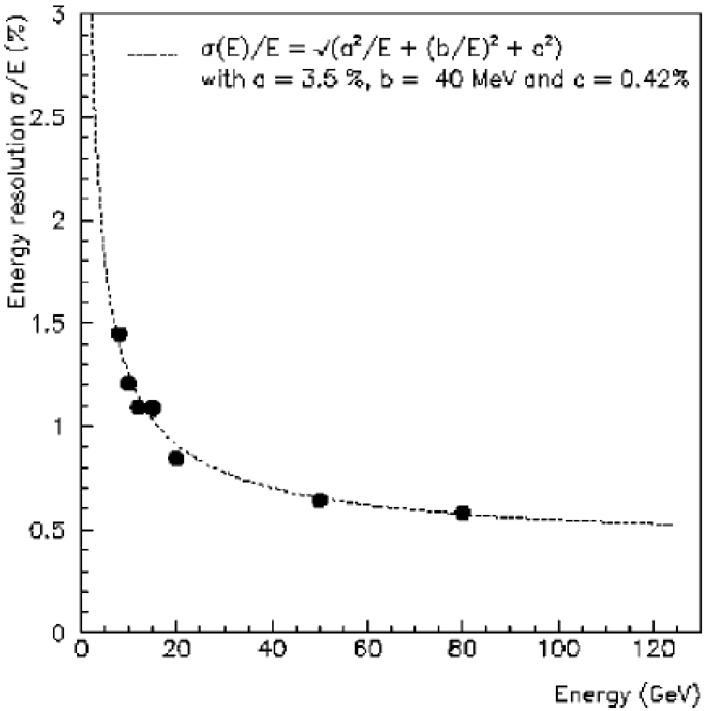


FIG. 3. Fractional electron energy resolution as a function of energy measured with a prototype of the NA48 liquid krypton electromagnetic calorimeter (NA48 Collaboration, 1995). The line is a fit to the experimental points with the form and the parameters indicated in the figure.

Plot taken from Fabjan, Gianotti, *„Calorimetry for particle physics“*, Rev.Mod.Phys. (2003) 75 - link

- The larger the incoming particle energy, the better the resolution
- Opposite trend with respect to spectrometers ($\sigma/p_T \propto p_T$)
- However, at very large energies...

Shower containment

Animations from an online EM shower simulator:
https://www.mpp.mpg.de/~menke/elss/home.shtml

Electrons hitting a 37x10x10 cm^3 lead-glass detector:

1 GeV

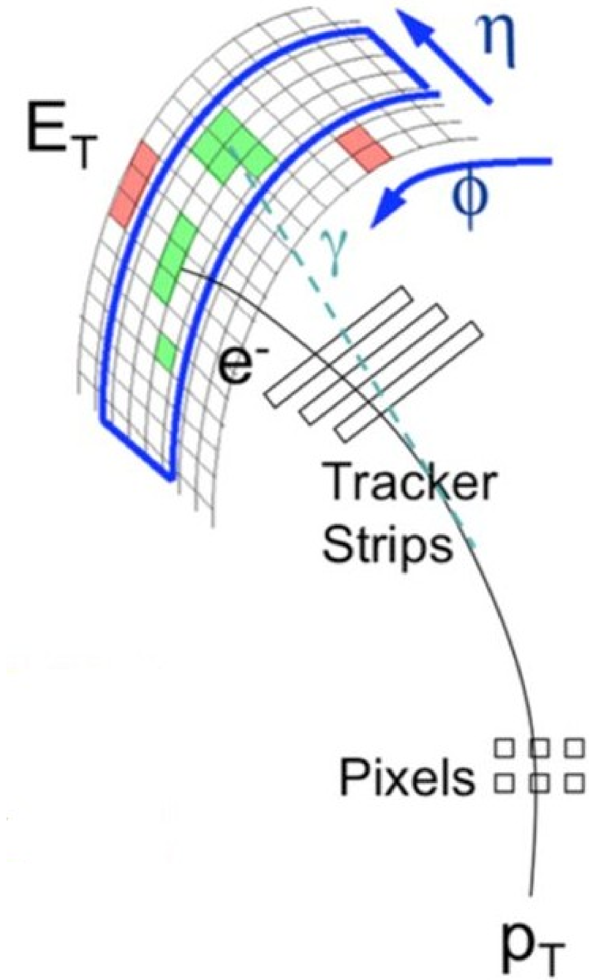
10 GeV

80 GeV

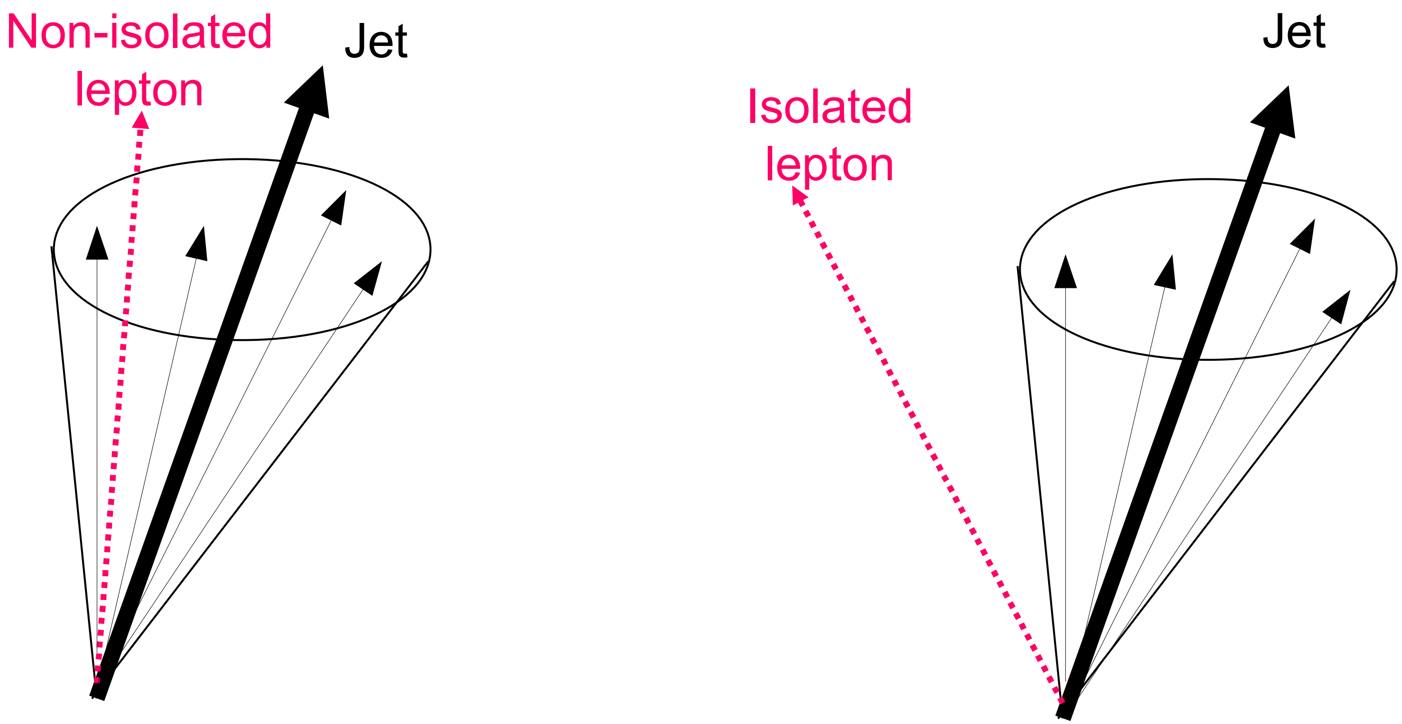
The higher the energy, the larger the probability that some particle from the shower is not contained in the detector.

Electron identification

- A particle is identified as an electron if:
  - Trajectory from the inner tracker matches a signal in the EM calorimeter with ~ the same energy; this rejects accidental overlaps (e.g., $\pi^{\pm}$ and γ)
  - The energy seen in the hadronic calorimeter is ~0, or at least much less than in the EM calorimeter; this rejects most hadrons (*Q: what about π^0?*)
  - Spatial shape of the signal cluster is consistent with a single particle radiating photons orthogonally to B field (narrow in z, broad in ϕ direction); this rejects accidental overlaps
- Measurement of the energy of the electron:
  - Weighted average of tracker info (special fit that considers bremsstrahlung) and EM calorimeter



Lepton (μ, e) isolation

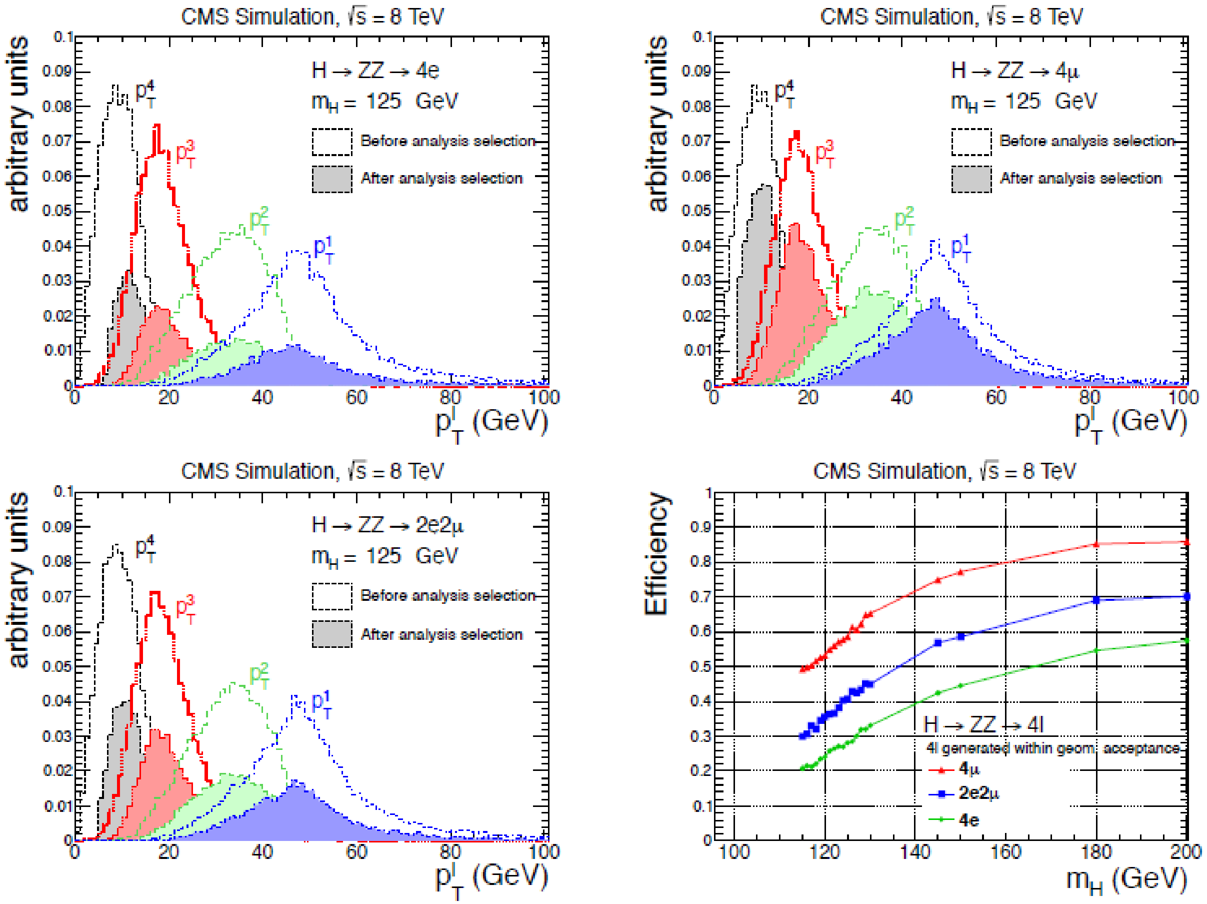


Leptons from Z or W decay are produced isolated, differently from fake leptons (punch-through hadrons faking muons, overlaps faking electrons) and real leptons produced by the quick decay of heavy quarks ($b,c \rightarrow e,\mu$), or by the slow decay of long-lived hadrons ($\pi,K \rightarrow \mu$), or by $\gamma \rightarrow e^+e^-$.
To quantify isolation, we sum the transverse momenta of particles nearby.

Event selection

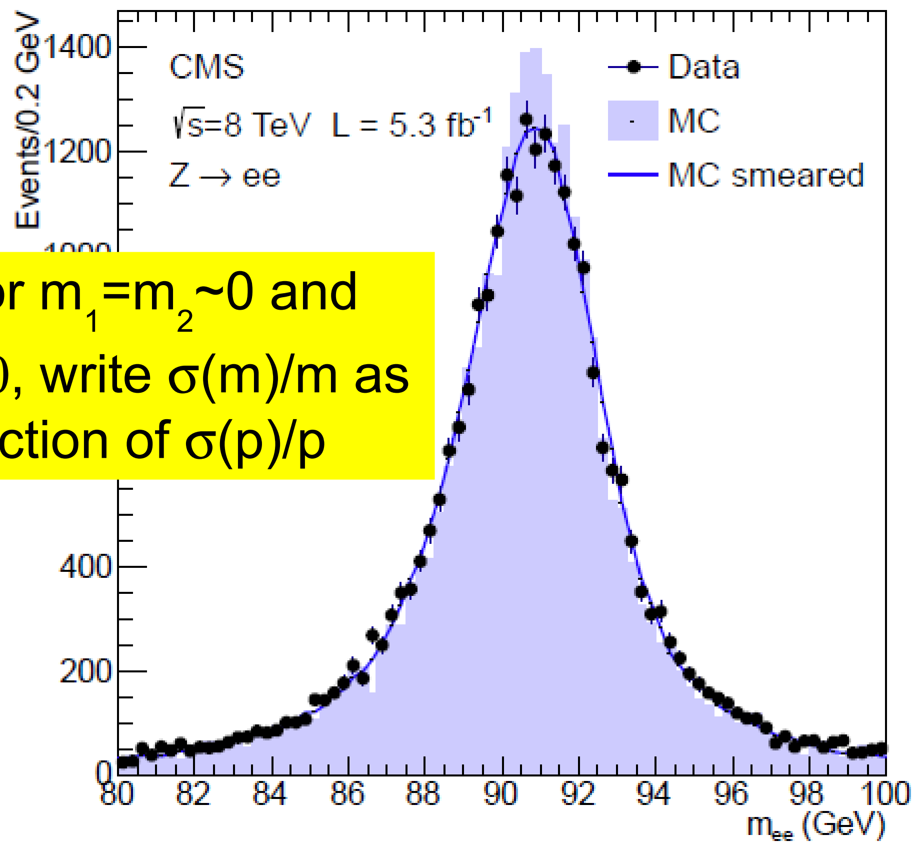
- Here and in the following, numbers and plots come from the CMS discovery paper (link) with some minor simplification
- Trigger: at least 2 leptons (ee, $\mu\mu$ or eμ) with p_T > 17 and 8 GeV
- Offline, at least 4 leptons must be present (4μ, 4e or 2μ2e), all isolated, with charge sum = 0 and p_T > 7 (e) or 5 (μ) GeV
- The two leading leptons must have p_T > 20 and 10 GeV (*Q: why is it a bit higher than the trigger threshold?*)
- These are very loose thresholds: p_T spectrum for leptons from Z decay, with Z at rest, extends up to $m_Z/2 \sim 45$ GeV ("Jacobian edge"), and even further if Z is boosted (as in our signal)

Lepton spectra in signal



Invariant mass of lepton pairs

$m \to m_1+m_2 \Rightarrow m^2=(E_1+E_2)^2-(\mathbf{p}_1+\mathbf{p}_2)^2 = m_1^2+ m_2^2 +2[(m_1^2+ p_1^2)^{1/2} (m_2^2+ p_2^2)^{1/2} - p_1p_2 \cos\alpha]$



Q: for $m_1=m_2\sim0$ and $\Delta\alpha/\alpha\sim0$, write $\sigma(m)/m$ as function of $\sigma(p)/p$

This analysis assumed that at least one Z is real. A priori, the other could be real or virtual (virtual if $M_H < 2M_Z$).

We call Z1 the candidate with mass closer to m_Z.

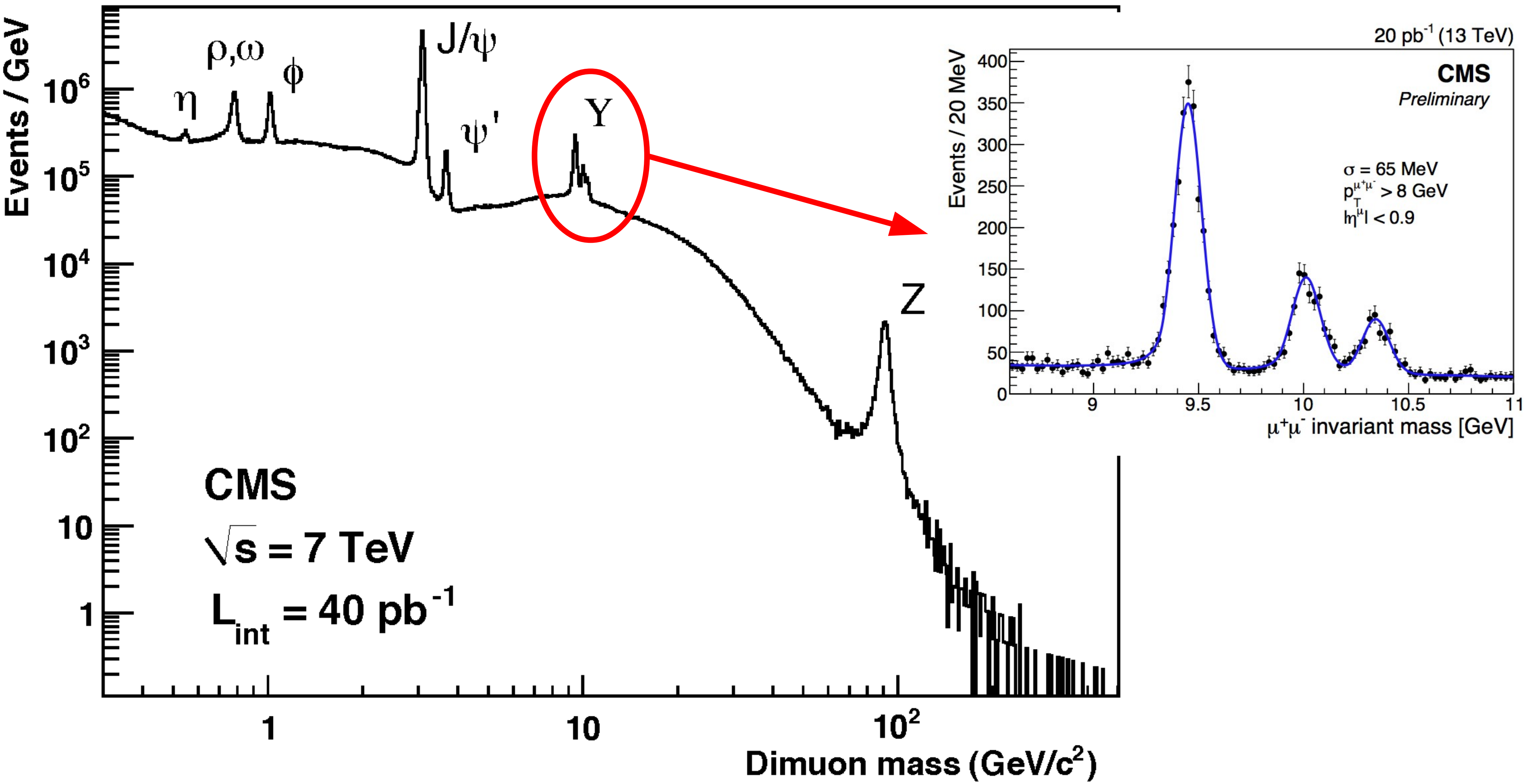
Selection:

40 GeV < m(Z1) < 120 GeV

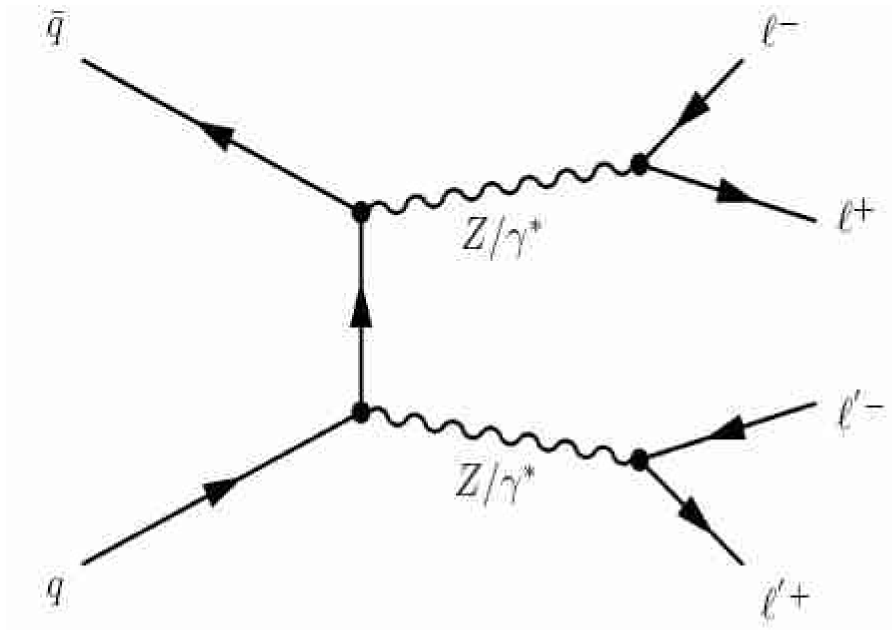
12 GeV < m(Z2) < 120 GeV

Q: why not less than 12 GeV?

The Drell-Yan spectrum

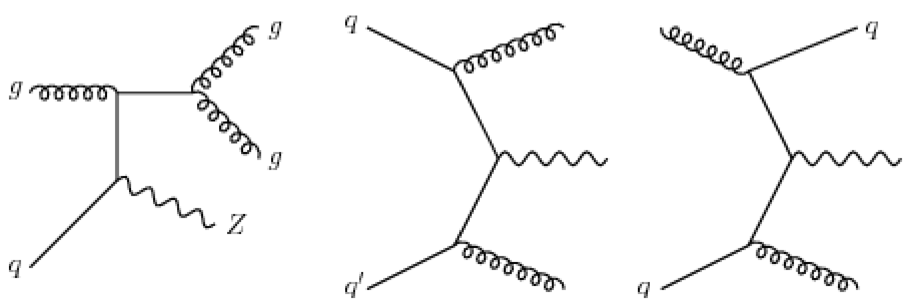


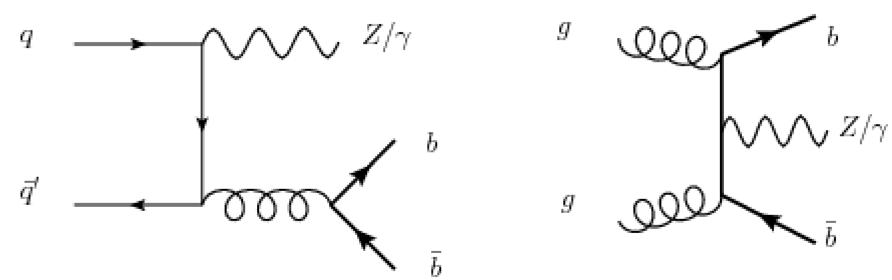
Z(*)Z* / Zγ* background



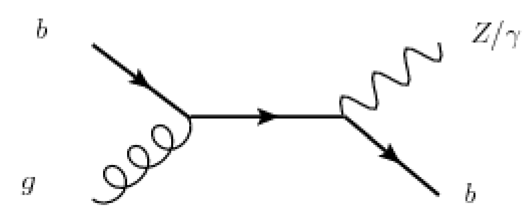
In our jargon, we say that this background is *irreducible* because it has the same final state particles as the signal; it cannot be reduced by improving the particle identification, differently from the backgrounds that I will illustrate in the next slides.

Z+X background





(a)

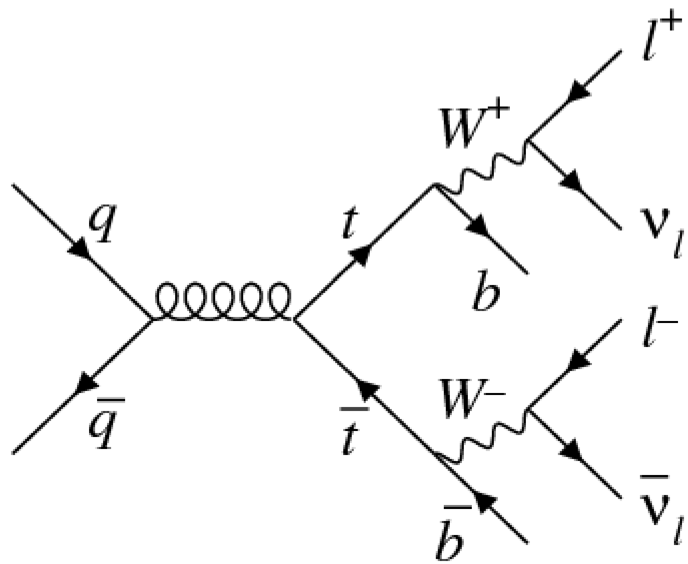


(b)

Z+jets: at LHC energies, the incoming partons radiate a lot of gluons, so it is very common to have energetic partonic jets accompanying the Z boson. Inside those jets there can be fake leptons, or π,K$\rightarrow\mu$ decays

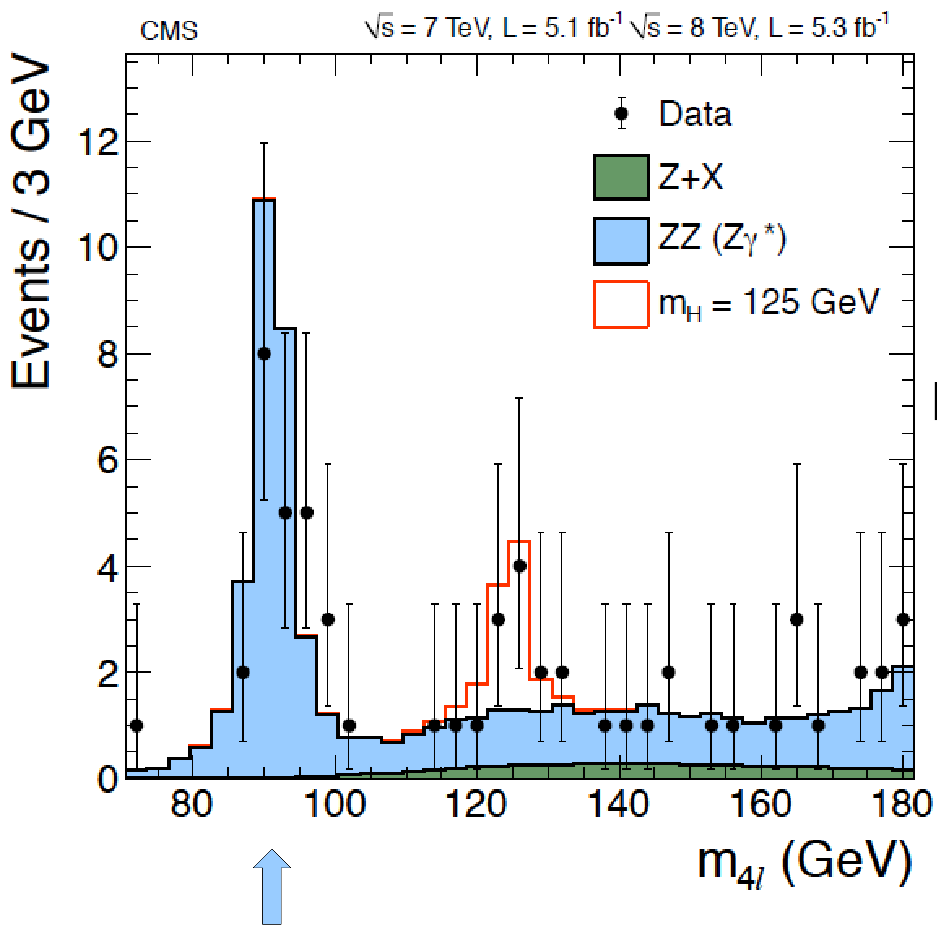
In a fraction of cases, the Z is produced in association with b or c, which decay 20% of times in e,μ

Top-quark background



Almost all top quarks decay into b quarks.
The b quark decays 20% of times in e,μ. This means ~40% probability of finding a lepton in a b-jet if one considers that almost all b decays go into c, and also the c quark decays 20% of times in e,μ.

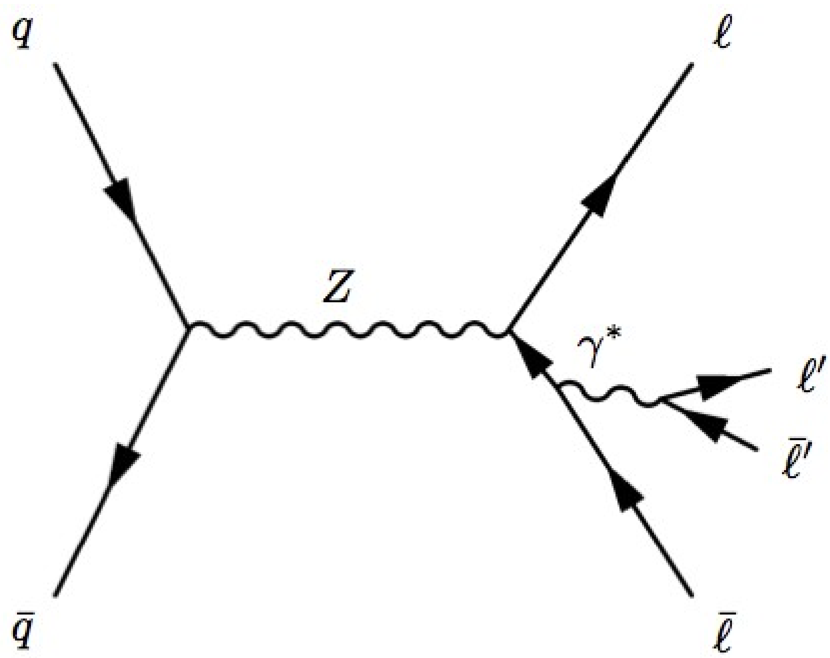
4l invariant mass (CMS 2012)



Note: top background is negligible, mostly thanks to isolation cuts

Q: what is this peak?

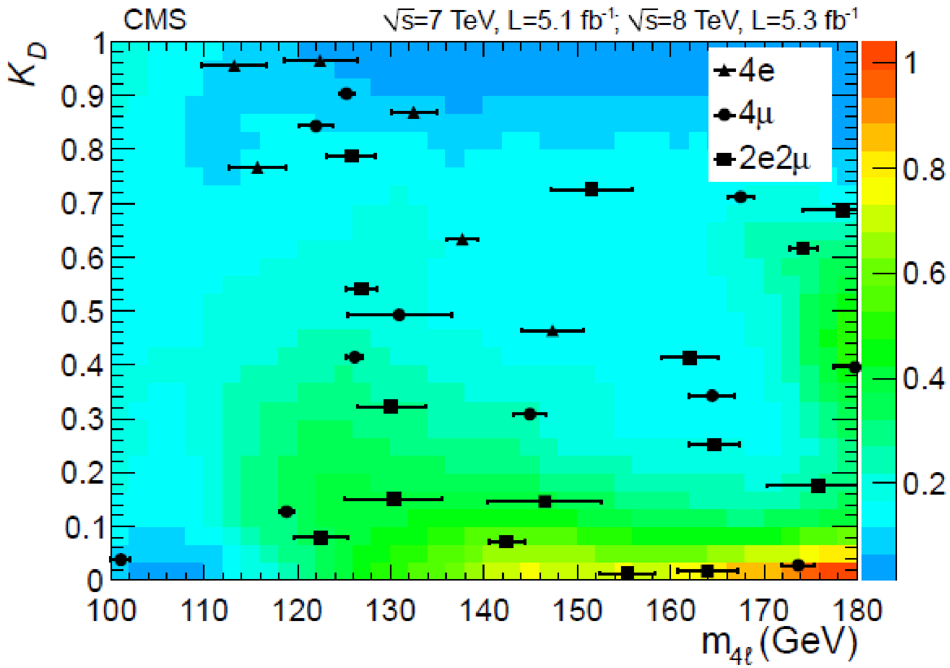
Z→4l background



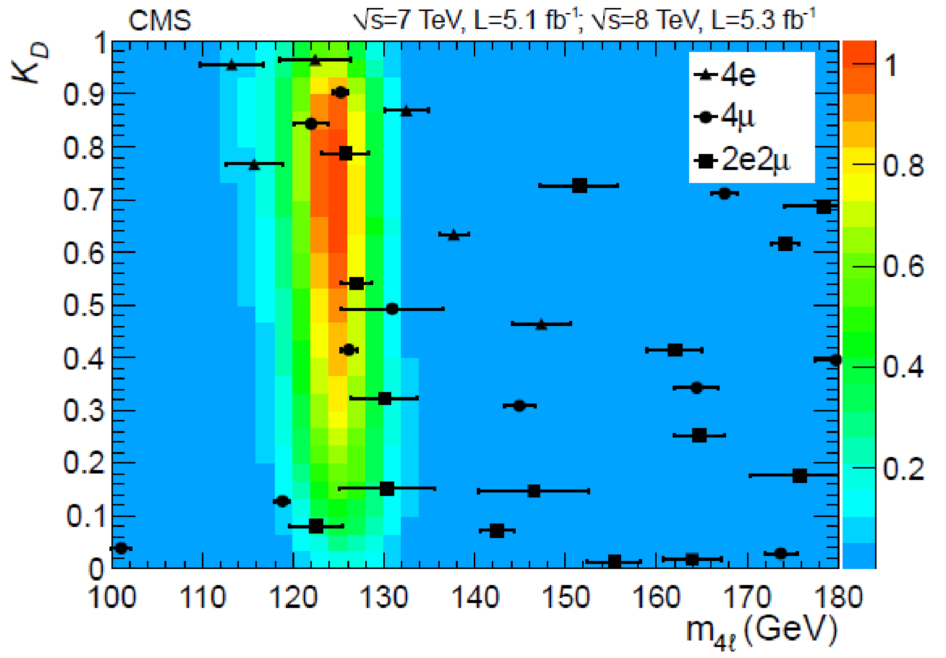
It is a very rare decay, only 1 out 4.5×10^{-6} Z's decay this way.
Studied in a dedicated publication:
CMS collaboration, JHEP 1212 (2012) 034

Kinematic discriminant

Basic property of the Higgs boson, as we saw in the first lesson, is to be a scalar ⇒ spin=0 ⇒ it decays isotropically.
A powerful discriminant is built by combining all independent angular distributions and Z1, Z2 invariant masses.

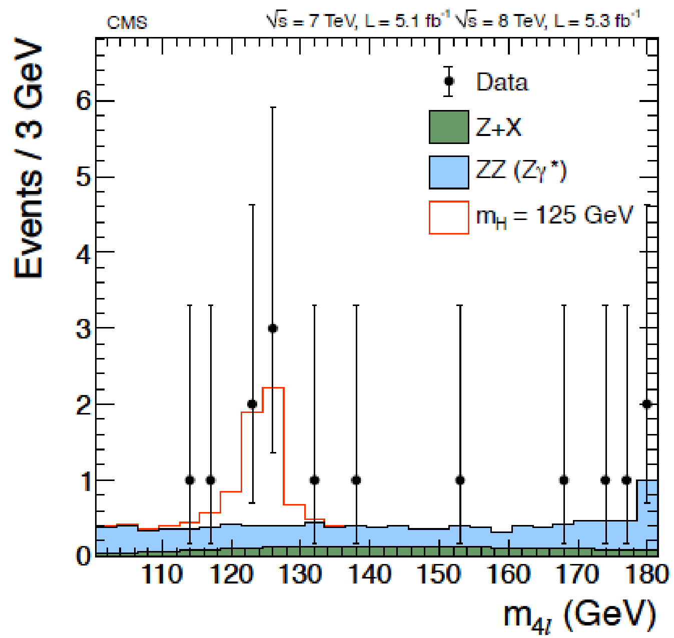


Data superimposed to background expectation



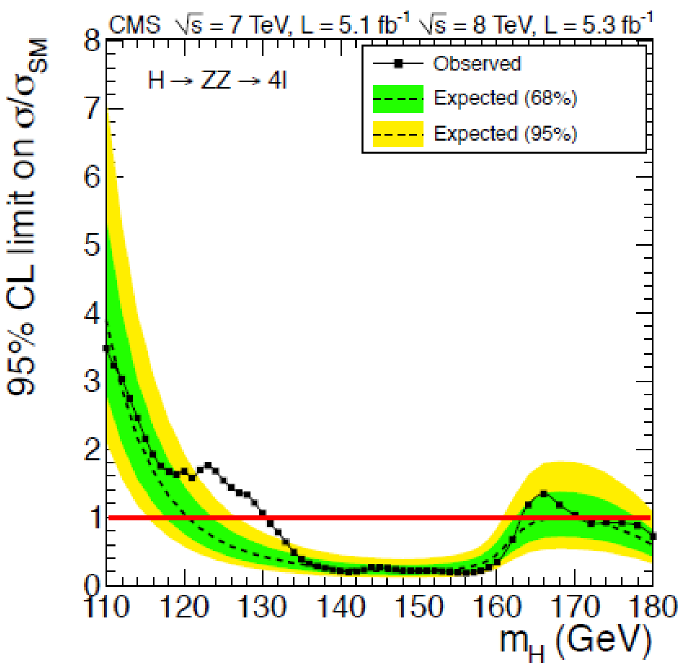
Data superimposed to Higgs @ 125 GeV expectation

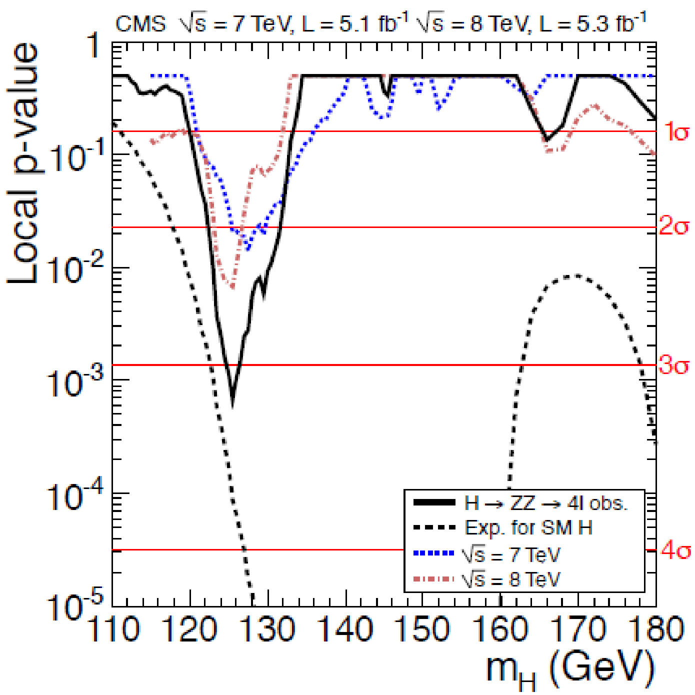
After kinematic discriminant cut



Cutting on the value of the kinematic discriminant (>0.5) confirms that the events around the peak are very signal-like

Fit result





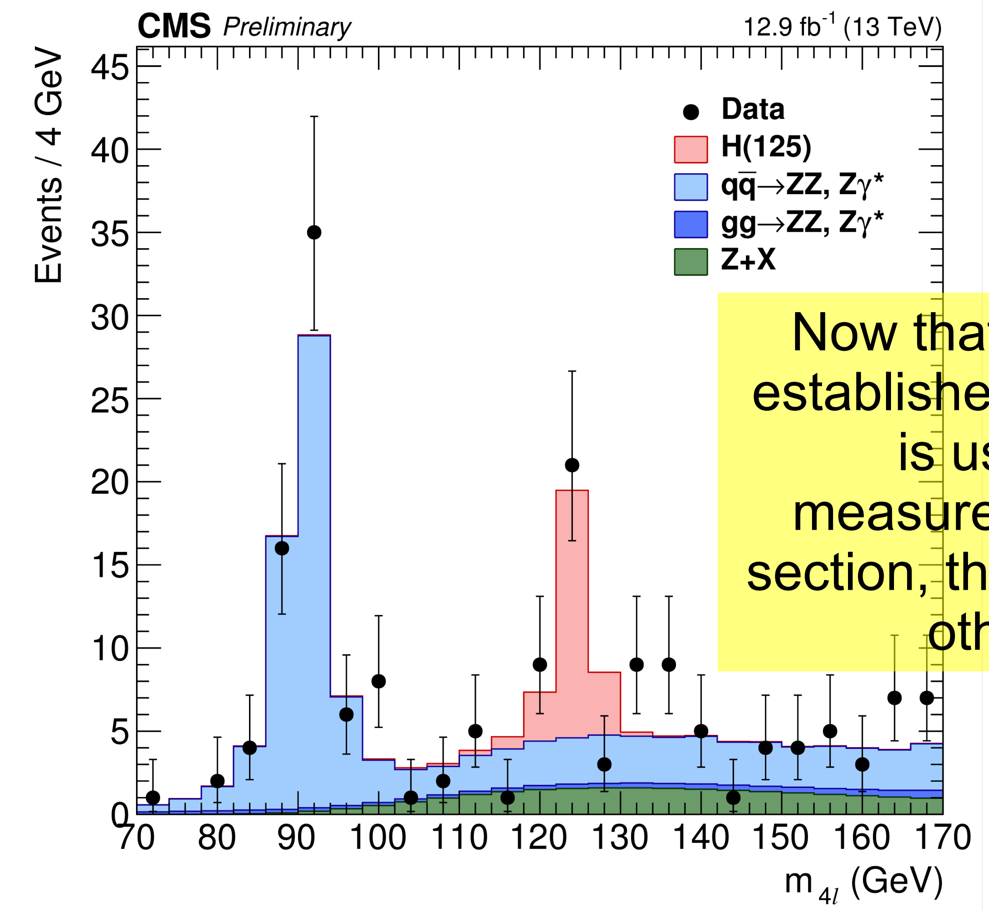
The p-value is the a-priori probability of observing an excess of data, with respect to the background-only hypothesis, at least equal to the one that we actually observed. Local p-value: at a given mass. Global p-value: taking into account the *Look Elsewhere Effect*.

Homework

Channel	4e	4μ	$2e2\mu$	Total
ZZ background	2.7 ± 0.3	5.7 ± 0.6	7.2 ± 0.8	15.6 ± 1.4
Z + X	$1.2^{+1.1}_{-0.8}$	$0.9^{+0.7}_{-0.6}$	$2.3^{+1.8}_{-1.4}$	$4.4^{+2.2}_{-1.7}$
All backgrounds ($110 < m_{4\ell} < 160\,\mathrm{GeV}$)	$3.9^{+1.1}_{-0.8}$	$6.6^{+0.9}_{-0.8}$	$9.5^{+2.0}_{-1.6}$	$20.0^{+3.2}_{-2.6}$
Observed ($110 < m_{4\ell} < 160\,\mathrm{GeV}$)	6	6	9	21
Expected Signal ($m_{\mathrm{H}} = 125\,\mathrm{GeV}$)	1.37 ± 0.44	2.75 ± 0.56	3.44 ± 0.81	7.6 ± 1.1
All backgrounds (signal region)	$0.71^{+0.20}_{-0.15}$	$1.25^{+0.15}_{-0.13}$	$1.83^{+0.36}_{-0.28}$	$3.79^{+0.47}_{-0.45}$
Observed (signal region)	1	3	5	9

- The real analysis made use of fairly complex statistical methods (likelihood fit with profiling of systematics), but you can use the table above for a *cut-and-count analysis*
  - Q1: estimate the significance (⇔ p-value) of the excess in the signal region, ignoring any systematic uncertainty
  - Q2: as above, for the signal expectation in the 125 GeV hypothesis
  - Q3: as above, assuming a 50% uncertainty on the sum of backgrounds
  - Q4: propose some method to decrease the background uncertainty

How things look like, today



Now that the Higgs is firmly established, the process H→4l is used for precise measurements of the cross section, the mass, the spin and other properties

Reference: CMS-HIG-16-033

Questions?

Caractérisation globale d'une collision hadronique, les variables cinématiques utilisées

$$p_T = p_\perp = \sqrt{p_x^2 + p_y^2} = p\sin\theta$$ **Moment transversal**

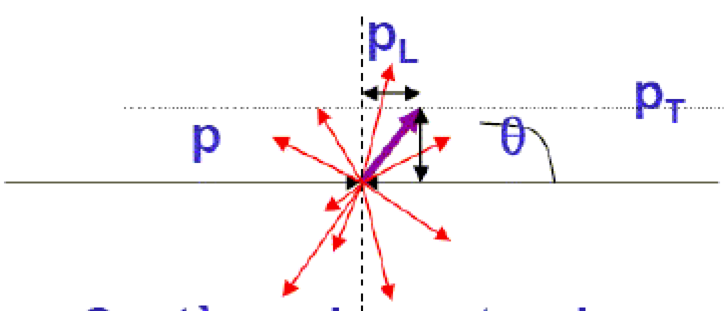
Section efficace invariant

$$E\frac{d^3\sigma}{dp^3} = E\frac{d^3\sigma}{dp_x dp_y dp_z} = \frac{1}{2\pi}\frac{d^2\sigma}{p_T dp_T d\left(p_L / E\right)} \sim \underbrace{F\left(p_T\right)F'\left(p_L\right)}$$

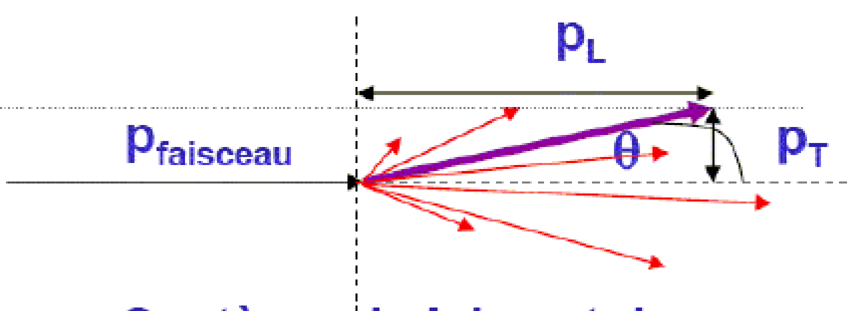
(Feynman scaling)

$$F\left(p_T\right) \sim e^{-bp_T} \; ; \; < p_T >_{\text{particules sécondaires}} \approx 0.3 - 0.5 \; GeV/c \; \approx \frac{\hbar}{R}$$

$$E_T = \sum_{i=\text{part.sécondaires}} E_i \sin\theta_i$$ **Energie transversal**



Système de centre de masse



Système de laboratoire

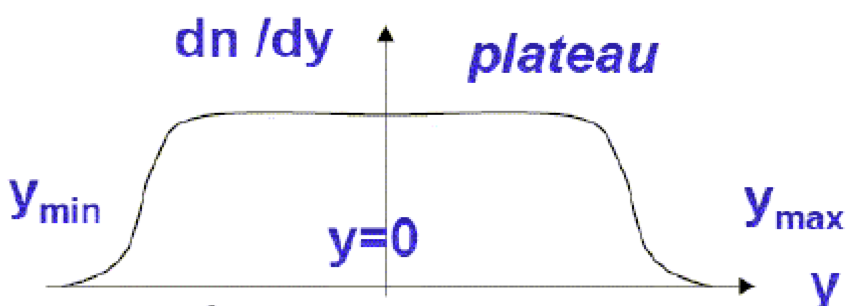
$$x_F = p_L / p_L^{\max} = p_L / \left(\sqrt{s}/2\right) \quad \text{(Feynman "}x\text{")}$$

Rapidité *y*, « invariante » de Lorentz

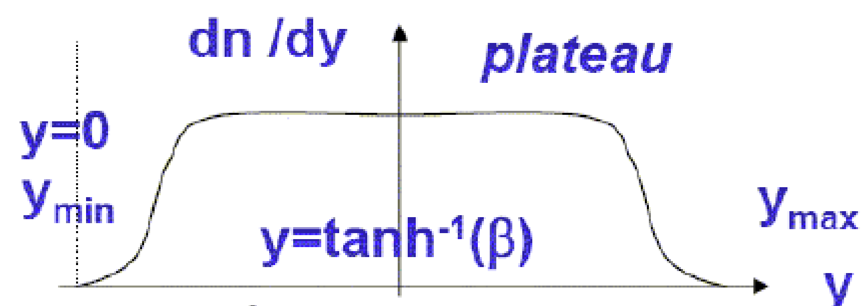
$$y = \frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right) \overset{\beta\to 1,\ m\to 0}{\approx} \eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

η, pseudo-rapidité

$$y \to y + \tanh^{-1}(\beta) \qquad y_{\max} = \frac{1}{2}\ln\left(\frac{s}{m^2+p_T^2}\right)$$



Système de centre de masse



Système de laboratoire

38.5.2. ***Inclusive reactions***: Choose some direction (usually the beam direction) for the z-axis; then the energy and momentum of a particle can be written as

$$E = m_T \cosh y \ , \ p_x \ , \ p_y \ , \ p_z = m_T \sinh y \ , \tag{38.35}$$

where m_T is the transverse mass

$$m_T^2 = m^2 + p_x^2 + p_y^2 \ , \tag{38.36}$$

and the rapidity y is defined by

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$$

$$= \ln \left(\frac{E + p_z}{m_T} \right) = \tanh^{-1} \left(\frac{p_z}{E} \right) \ . \tag{38.37}$$

Under a boost in the z-direction to a frame with velocity β, $y \to y - \tanh^{-1} \beta$. Hence the shape of the rapidity distribution dN/dy is invariant. The invariant cross section may also be rewritten

$$E \frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi \, dy \, p_T \, dp_T} \Longrightarrow \frac{d^2\sigma}{\pi \, dy \, d(p_T^2)} \ . \tag{38.38}$$

For $p \gg m$, the rapidity [Eq. (38.37)] may be expanded to obtain

$$y = \frac{1}{2} \ln \frac{\cos^2(\theta/2) + m^2/4p^2 + \ldots}{\sin^2(\theta/2) + m^2/4p^2 + \ldots}$$

$$\approx -\ln \tan(\theta/2) \equiv \eta \tag{38.42}$$

where $\cos\theta = p_z/p$. The pseudorapidity η defined by the second line is approximately equal to the rapidity y for $p \gg m$ and $\theta \gg 1/\gamma$, and in any case can be measured when the mass and momentum of the particle is unknown. From the definition one can obtain the identities

$$\sinh\eta = \cot\theta \ , \quad \cosh\eta = 1/\sin\theta \ , \quad \tanh\eta = \cos\theta \ . \tag{38.43}$$

EM interactions

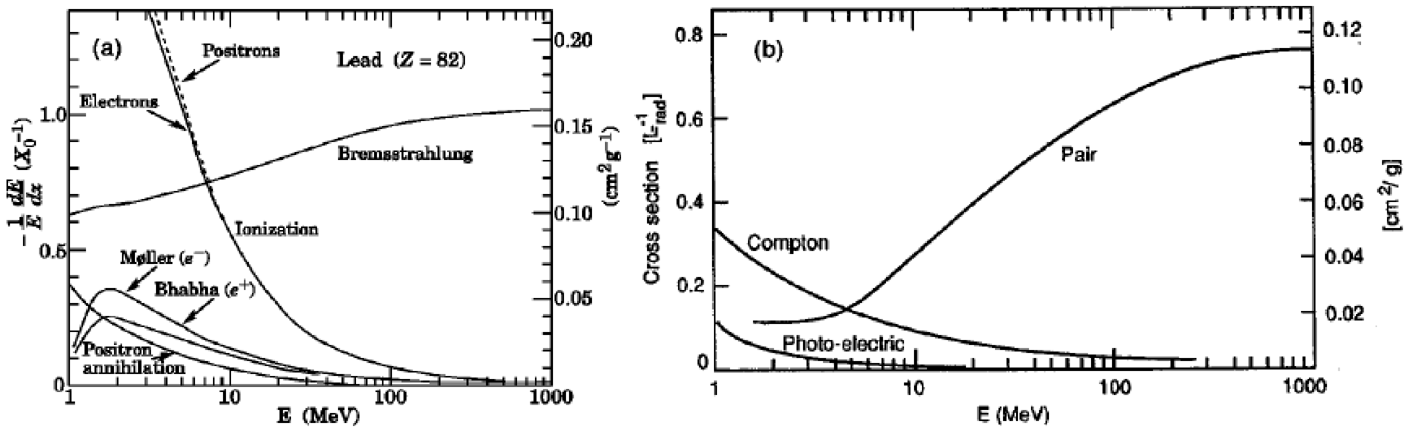


FIG. 1. (a) Fractional energy lost in lead by electrons and positrons as a function of energy (Particle Data Group, 2002). (b) Photon interaction cross section in lead as a function of energy (Fabjan, 1987).

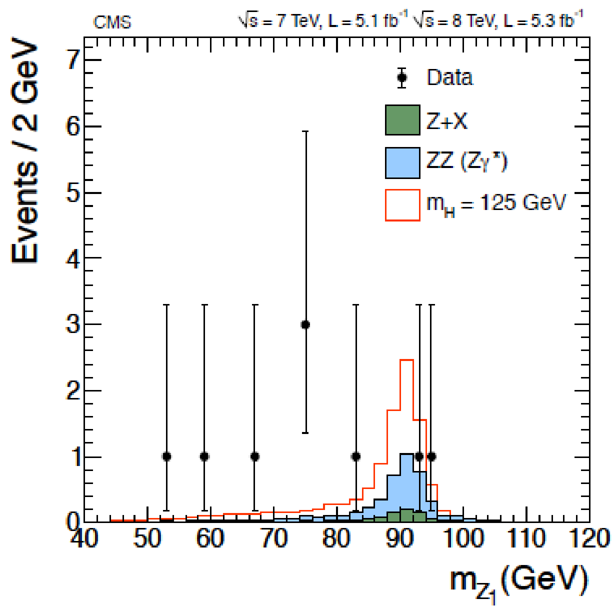
To know more on calorimetry:

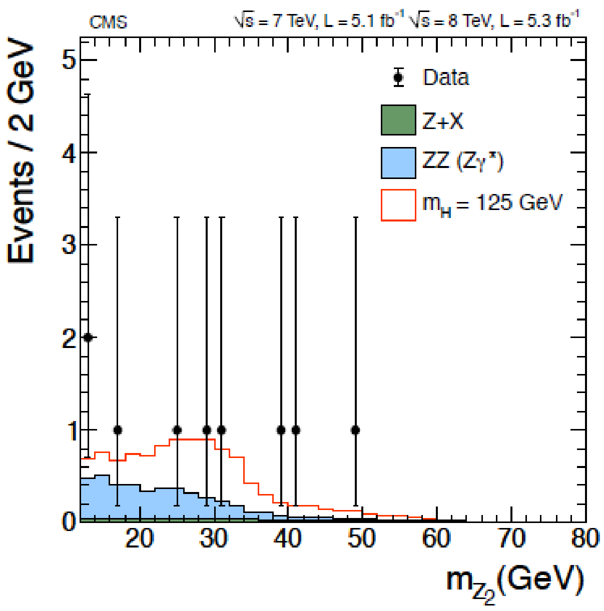
REVIEWS OF MODERN PHYSICS, VOLUME 75, OCTOBER 2003

Calorimetry for particle physics

Christian W. Fabjan and Fabiola Gianotti

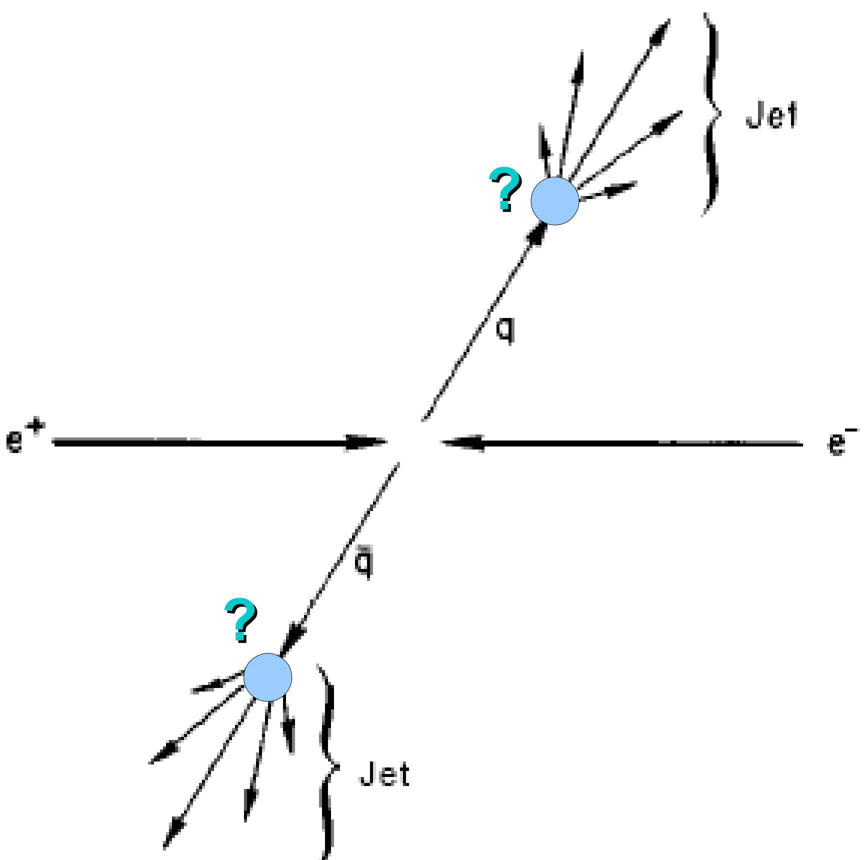
2l invariant masses (CMS 2012)

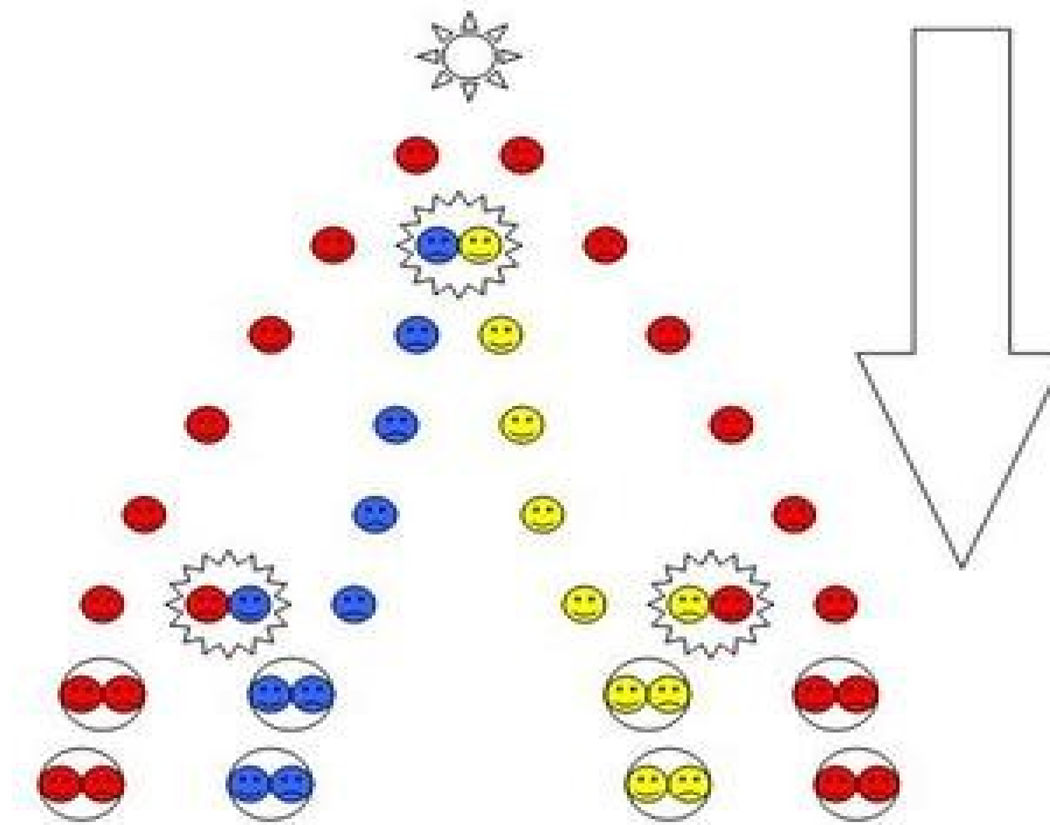




"Fragmentation" (also known as "hadronization")

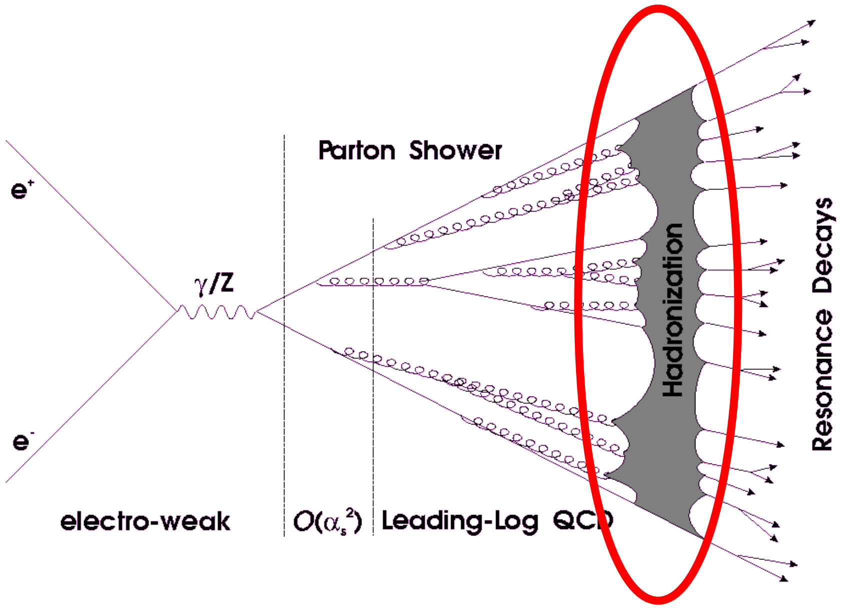
- As you know, you can't observe quarks directly
- QCD explanation: the attraction <u>increases</u> with r, so at some point the potential energy of the system is larger that $2m_q$



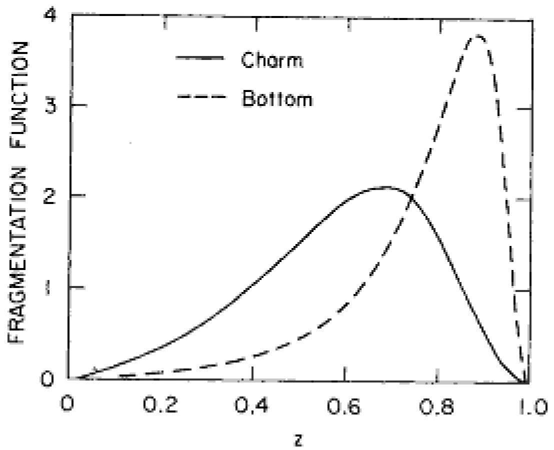


Credit for this picture: T.Dorigo

Fragmentation function



$$z = \frac{(E + p_{||})_{\text{hadron}}}{(E + p_{||})_{\text{quark}}}$$



$$x_B = \frac{E_{\text{hadron}}}{E_{\text{beam}}}$$

Peterson's function for heavy quarks:

$$D_Q^H(z) = \frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2} \, .$$