Particle Physics II
(LPHY2133)

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Basic info for this course

- Spoken language: as this is an International Master, you can choose between French and English
  - Anyway, all my slides will be in English, and I will recommend articles and books in English
- This course is split in two parts:
  - February-March: **the Higgs boson**, mostly from the experimental point of view
  - April-May: **detectors for particle physics** (by Prof. Krzysztof Piotrzkowski)
- Webpage for the first part of the course:
  - http://cern.ch/andrea.giammanco/particules2017
  - You will find all slides, and more material

References for my part of the course

- In growing order of difficulty:
- **D. Perkins, "Introduction to High Energy Physics"**
  - Chapter 8, "Electroweak interactions and the Standard Model"
- **V. Barger, R. Phillips, "Collider Physics"**
  - Chapter 2, "The Standard Electroweak Gauge Model"
  - Chapter 12, "Higgs Boson"
- **J. Gunion et al., "The Higgs Hunter's Guide"**
  - I requested a copy to be in our library
- **Particle Data Group, "Review of Particle Physics"**
  - Free for download here: http://pdg.lbl.gov/
- Some research papers that I will refer through the course

Section 1

Theorethical introduction / reminder

My apologies in advance if what I will discuss today is shown to you for the Nth time; but I need to ensure that some specific aspects (which may not be the most interesting for other teachers with different pedagogical goals) are highlighted, as I will then need those concepts later as the basis for some *experimental* discussion (which is my own pedagogical goal).

Equations of motion

For a scalar field representing particles of mass m, in the absence of any interaction, the equation of motion is the Klein-Gordon equation:

$$\left(\partial^{\mu}\partial_{\mu}+m^{2}\right)\phi=0$$

For a fermionic field, the equation of motion is the Dirac equation:

$$\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi=0$$

This field is mathematically described by a spinor.

Lagrange equation

Lagrangian: difference between kinetic and potential energy (L=K-V)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

In quantum field theory, and after defining L as a Lagrangian *density*:

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad \textbf{(Eq.1)}$$

Free massive scalar field

Lagrangian density of a free (i.e., no potential) massive scalar field:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \quad \textbf{(Eq.2)}$$

From equations 1 and 2 one gets the equation of motion, that turns out to be the Klein-Gordon equation:

$$(\partial^\mu \partial_\mu + m^2)\phi = 0 \quad \textbf{(Eq.3)}$$

(In fact, eq.2 was historically chosen such to yield the K-G equation, which is well motivated from symmetry arguments!)

Self-interacting scalar field

The previous slide described a boring universe. Make it slightly less boring: the field interacts with itself (i.e., its quanta can scatter, annihilate, radiate each other)

A priori the **potential** can be anything; let's impose some physical constraints:

- Symmetry between $+\phi$ and $-\phi \Rightarrow$ the potential has to be a function of ϕ^2
  - (in reality the Higgs field is a SU(2) doublet $\Rightarrow \phi$ is 2-dimensional, but the argument is the same: no physical effect should depend on arbitrary direction in this 2-d space)
- $V(\phi^2)$ has a smooth behavior around the $\phi^2=0$ value $\Rightarrow$ can be expanded polynomially $\Rightarrow V(\phi^2)=(1/2)a\phi^2+(1/4)b\phi^4+(1/6)c\phi^6+...$
  - (the fractions are just a convention: what really matters is the eq.of motion, that you get by taking the derivative; it is annoying to carry those numbers around, so we put them at the denominator such that they cancel out in deriving the equation of motion)
- If V can be written as $V(\phi^2) = k\phi^2 + W(\phi^2)$, the quadratic term behaves as a mass term, because it gives a term proportional to ϕ in the eq. of motion, with inertial mass $\mu=2\sqrt{k}$; so let's just write $V(\phi^2) = (1/2)\mu^2\phi^2 + W(\phi^2)$ from now on
- It turns out (beyond the scope of this lecture) that ϕ^6 and larger powers give a non-renormalizable lagrangian $\Rightarrow$ we are left with $V(\phi^2) = (1/2)\mu^2\phi^2 + (1/4)\lambda\phi^4$

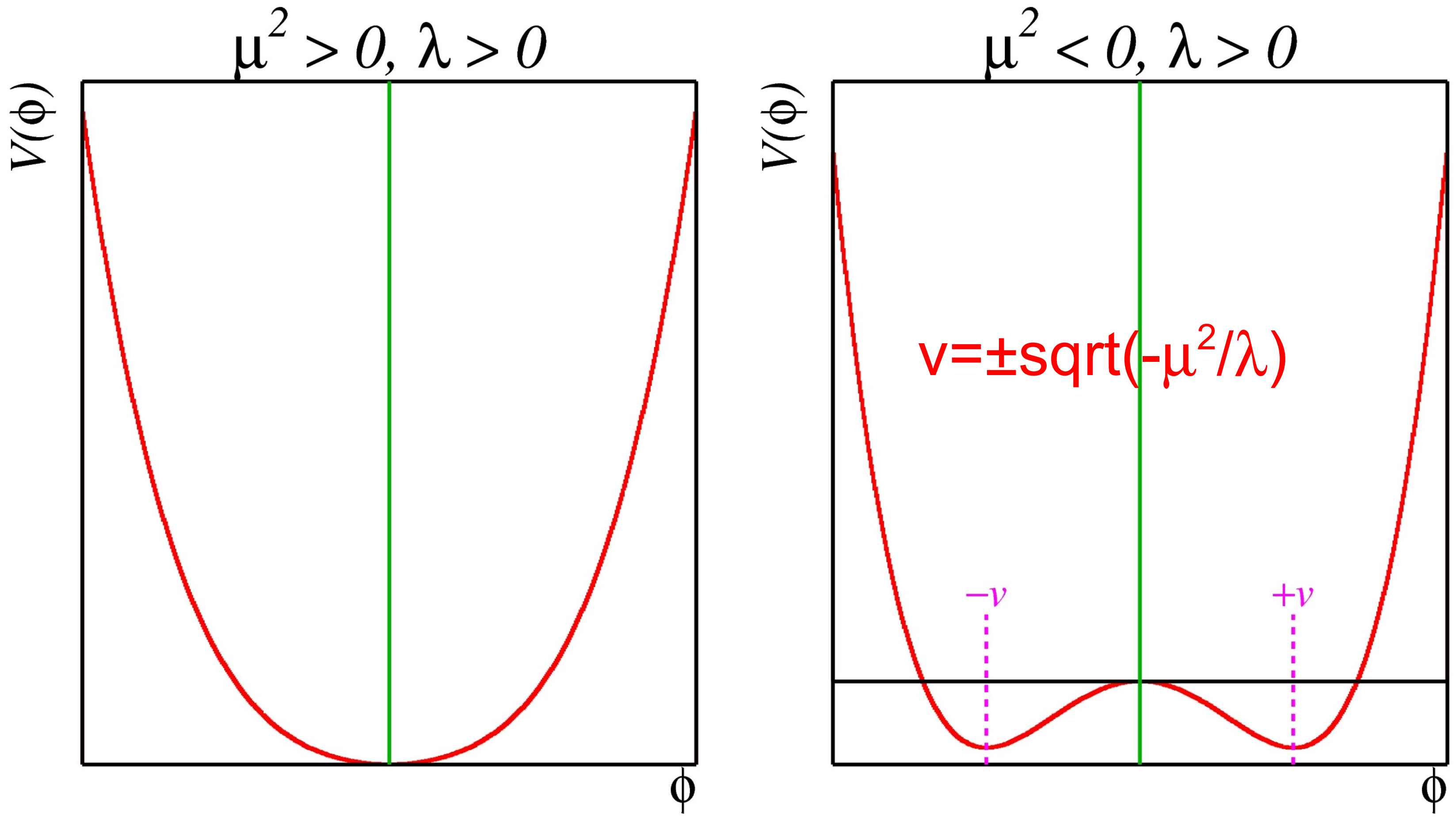
What is the ground state (aka *vacuum*) of this simple theory?

- In classical physics, all minima of V are stable states, and the lowest minimum is called "ground state"
- In quantum mechanics, tunnel effect causes all other minima to "decay" to the ground state
- As you learned when you were younger, a minimum of f(x) is defined by df/dx=0 and $d^2f/dx^2>0$
- Deriving from $(1/2)\mu^2\phi^2+(1/4)\lambda\phi^4$ (1-dimensional ϕ) we get the 3rd-degree equation $\phi(\mu^2+\lambda\phi^2)=0$
- It has 3 solutions: $\phi_{1,2}=\pm\text{sqrt}(-\mu^2/\lambda)$ and $\phi_3=0$

What is the ground state (aka *vacuum*) of this simple theory?

- It has 3 solutions: $\phi_{1,2}=\pm\mathrm{sqrt}(-\mu^2/\lambda)$ and $\phi_3=0$; which one is the ground state?
- You immediately notice that $\phi_{1,2}$ are imaginary (non-physical) if the argument of the square root is negative, and in that case the only solution is $\phi_3=0$
- Note: λ can not be negative, otherwise V is monotonically decreasing, $\phi_3=0$ is its maximum and there is no minimum

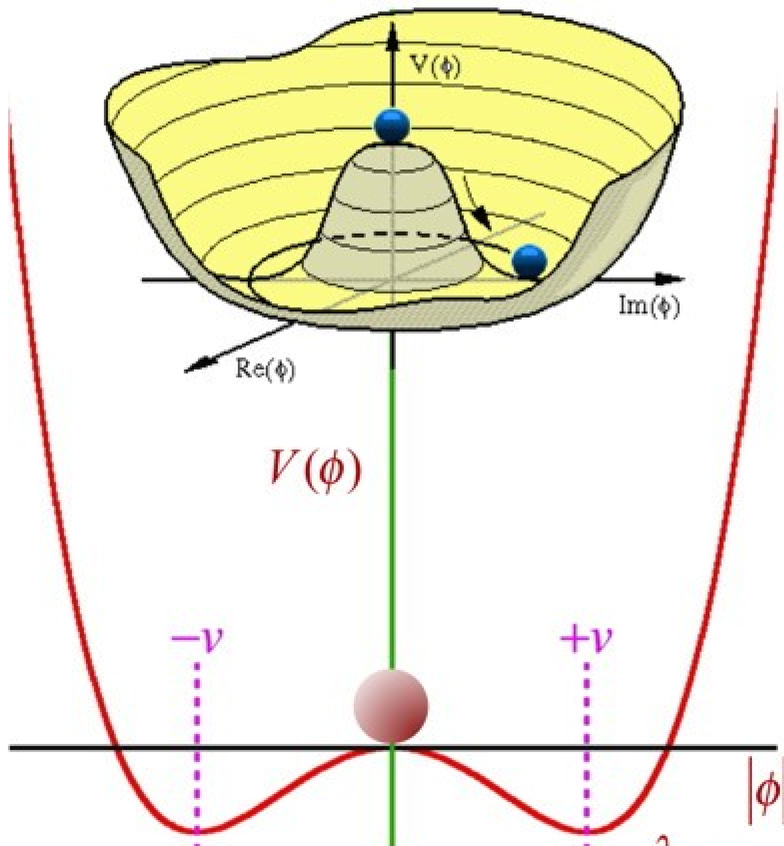
The Higgs self-interaction potential



Pictures from https://web2.ph.utexas.edu/~coker2/index.files/gaugef.htm

(Note: imaginary mass term → the quantum of this field is a tachyon; Is it a problem?)

In 2D (for reasons we will see later)



$$V(\phi) = \tfrac{1}{2}\mu^2\phi^\dagger\phi + \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2$$

Groundstate at $|\phi_0| = \sqrt{\frac{-\mu^2}{\lambda}} \equiv \upsilon$

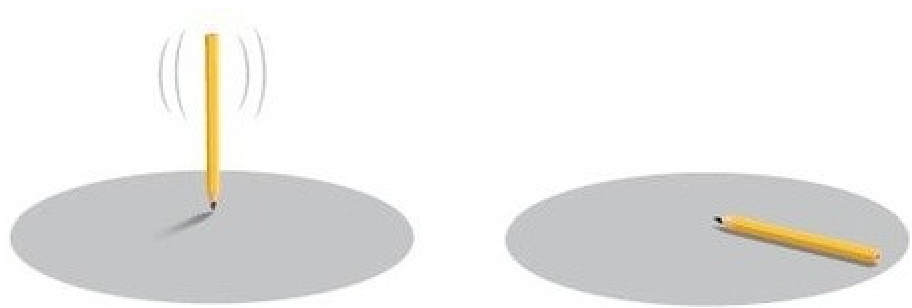
$$|\phi| = \sqrt{\phi^\dagger\phi} = \sqrt{\phi^{+\dagger}\phi^+ + \phi^{0\dagger}\phi^0}$$

$$V(\phi_0) = -\frac{\lambda}{4}\upsilon^4$$

The constant v is also called Vacuum Expectation Value

Pictures from https://web2.ph.utexas.edu/~coker2/index.files/gaugef.htm

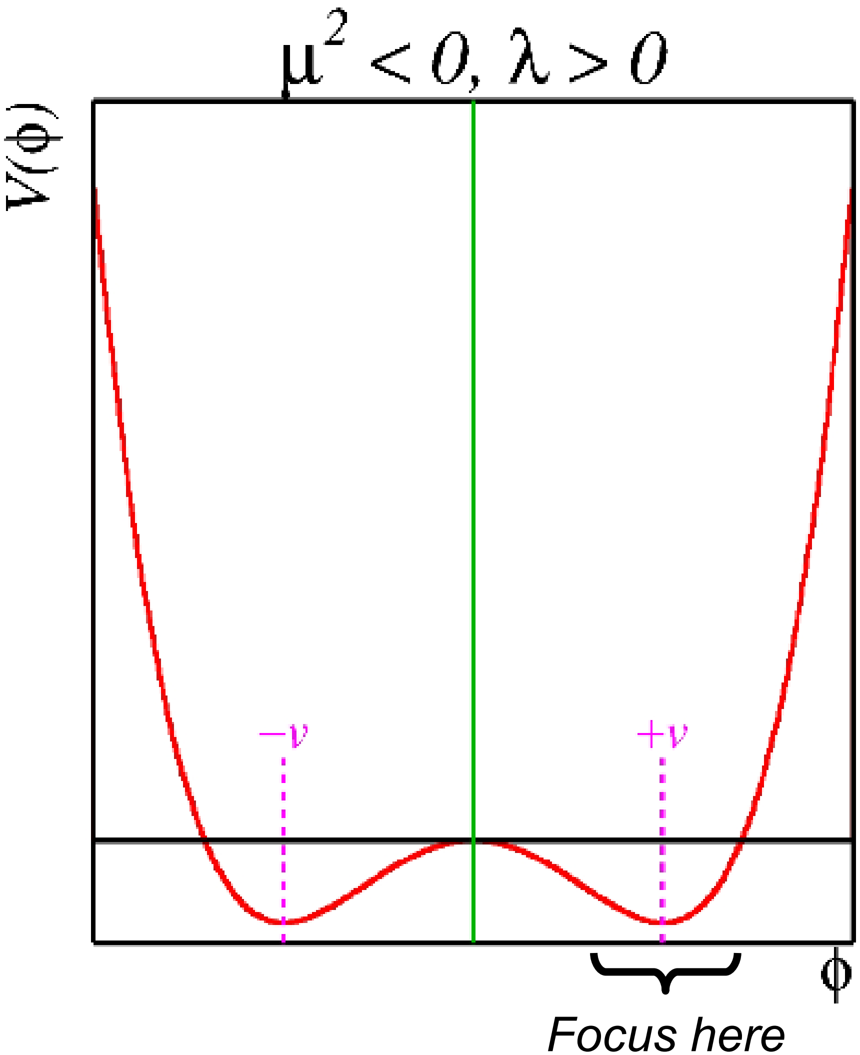
Analogy



The field has a symmetry, but the stable states are not symmetric.

A small initial perturbation forces the system to collapse in a final state randomly, among a continuum of states with the same energy.

The Brout-Englert-Higgs field close to the VEV



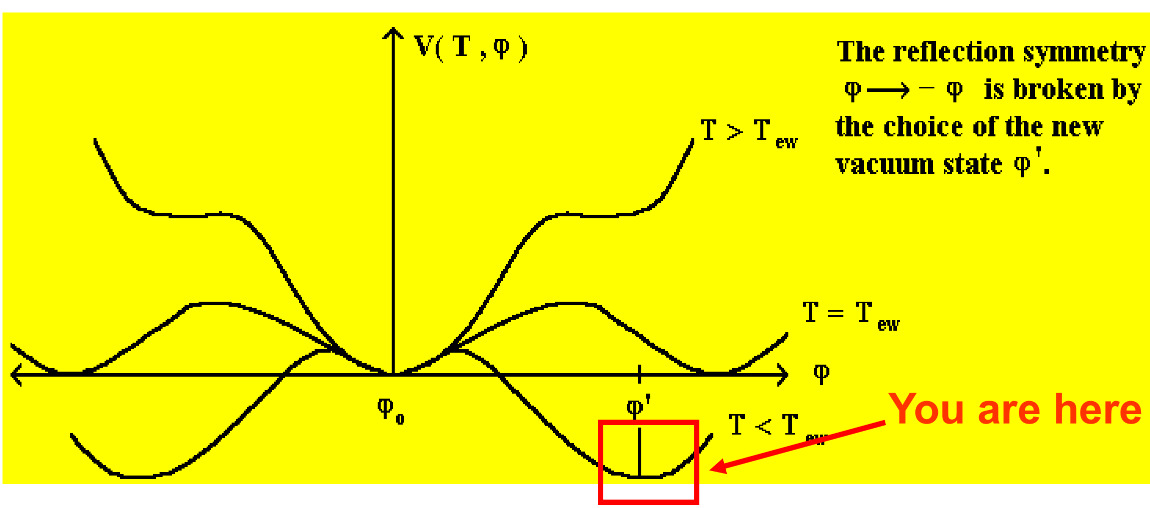
- Let's go back to the 1D model for illustration purposes
- There are two ground states; the system will spontaneously relax to one of them
- Write $\phi(x,y,z,t) = v + h(x,y,z,t)$ and you get:

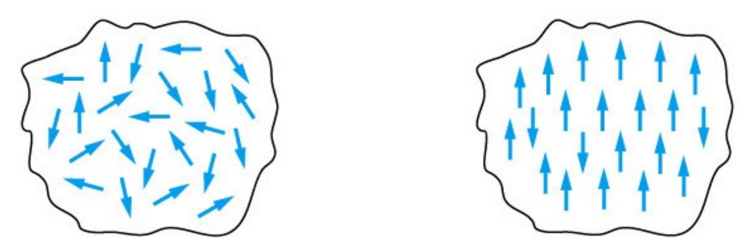
$$L = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - (\lambda v h^3 + \frac{1}{4}\lambda h^4) + const.$$

- Very similar to the original lagrangian, but this one has a 3[rd] power in the field, so it is less symmetric (can be seen also by eye in the picture of the potential)

Spontaneous Symmetry Breaking

- Basic idea: the lagrangian of a theory can have a symmetry that is not a symmetry of the ground state
- At low energy we are close to the ground state, and only at very high energy we can notice the symmetry
- Several other examples in Physics, e.g., ferromagnetism versus temperature:





$T > T_c$
perfect isotropy

$T < T_c$
one direction prevails
(but it is not special!)

The Higgs particle

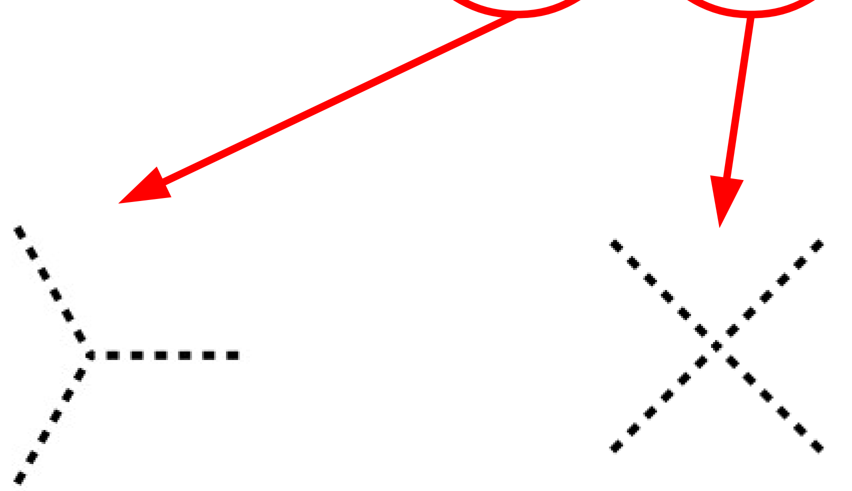
$$L = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \left(\lambda v h^3 + \frac{1}{4}\lambda h^4\right) + const.$$

- Mass term for the new field h(x,y,z,t)
- This time it is real and positive, so it is actually physical: $m_H = \text{sqrt}(2\lambda v^2) = \text{sqrt}(-2\mu^2)$
- We call **Higgs particle** the quantum of the h field, which is more convenient to use than the ϕ field when we want to study the physical effects
- The ϕ field is more convenient to use when we want to see the symmetries of the lagrangian at first sight

- Made of terms in v^2 and v^4 with no dependence on the field
- Constant terms in the lagrangian have no physical effects: what matters is the eq.of motion, that you get by taking the derivative

The Higgs self-interactions

$$L = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \left(\lambda v h^3 + \frac{1}{4}\lambda h^4\right) + const.$$



Reactions of the kind HH→H, H→HH and HH→HH are possible, although we didn't observe them yet.

Their observation will provide a measurement of the fundamental parameter λ.

Free massive fermion field (spinor)

Lagrangian density of a free (i.e., no potential) massive spinor:

$$L = i\,\bar{\psi}\,\partial\!\!\!/\,\psi - m\,\bar{\psi}\,\psi \qquad \textbf{(Eq.2')}$$

From equations 1 and 2' one gets the equation of motion, that turns out to be the Dirac equation:

$$\left(i\,\partial\!\!\!/ - m\right)\psi = 0 \qquad \textbf{(Eq.3')}$$

(Note the slight change of notation: from now on, to simplify the formulae, I am using the *Feynman slash*: $a\!\!\!/ \equiv a_\mu \gamma^\mu$)

Conservation laws for spinors

$$L = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi$$

Note that one can also use a lagrangian to quickly check if some quantities are conserved.

For example in this case you see that for each ψ (create a fermion or destroy an antifermion) you always have a $\bar{\psi}$ (destroy a fermion or create an antifermion).

So the number of fermions minus antifermions is conserved.

We can also have lagrangian terms of the kind $\bar{\psi}_\beta X \psi_\alpha$, where X is some operator that mediates the transition between particle α and particle β.

This happens for example in weak interactions, as we will see soon.

Gauge theories

$$L = i\,\bar{\psi}\,\partial\!\!\!/\,\psi - m\,\bar{\psi}\,\psi$$

An obvious symmetry of this Lagrangian is that a global phase rotation has no effect on it:

$$\psi(x) \rightarrow e^{+i\,\theta}\,\psi(x)$$
$$\bar{\psi}(x) \rightarrow e^{-i\,\theta}\,\bar{\psi}(x)$$

The gauge principle states that this must be true also for arbitrary *local* phase transformations, i.e., where θ is a function of space-time

In U(1)

The lagrangians that we have seen so far are not gauge-invariant; but at least for m=0 (we will see m>0 later) they become gauge invariant if you replace the partial derivative by the covariant derivative:

$$D_\mu = \partial_\mu - i\, g\, A_\mu \quad \text{with rule} \quad A_\mu \rightarrow A_\mu + \partial_\mu \theta$$

Here A is a long-range field and it is a 4-vector (index μ); g is a constant that represents the strength of the coupling between A and the fields to which you apply the operator D (example: in QED, g=e)

Plugging D in Eq.2', for example, causes the appearance of a **force** term that connects the spinor ψ and the new field A, with strength g:

$$L_{int.} = -i\, g\, \bar{\psi} \not{A} \psi$$

From lagrangian to vertex

$$L_{QED} = -i e \bar{\psi} \not{A} \psi$$

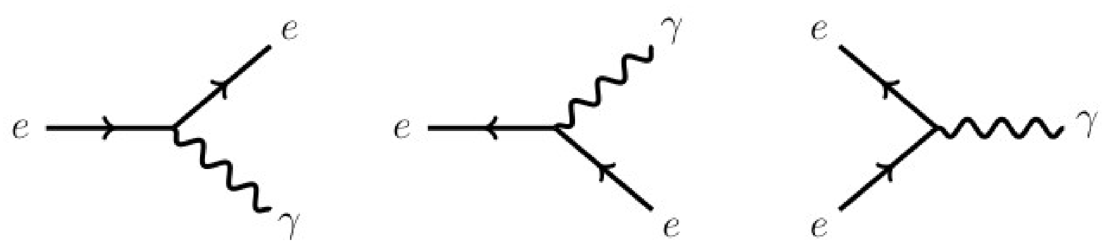


Image taken (with typo fixed) from:
http://www.quantumdiaries.org/2010/03/07/more-feynman-diagrams-momentum-conservation/

Mass of the force carrier

The new vectorial field A has also a kinetic energy term (we will not write it here), so it represents a physical particle. Can it have also a mass term?

In general, a mass term for a vectorial field must be written:

$$L_{mass} = \frac{1}{2} m^2 A_\mu A^\mu$$

<u>*This is not gauge-invariant*</u>; verify by applying the gauge transformation:

$$A_\mu \to A_\mu + \partial_\mu \theta$$

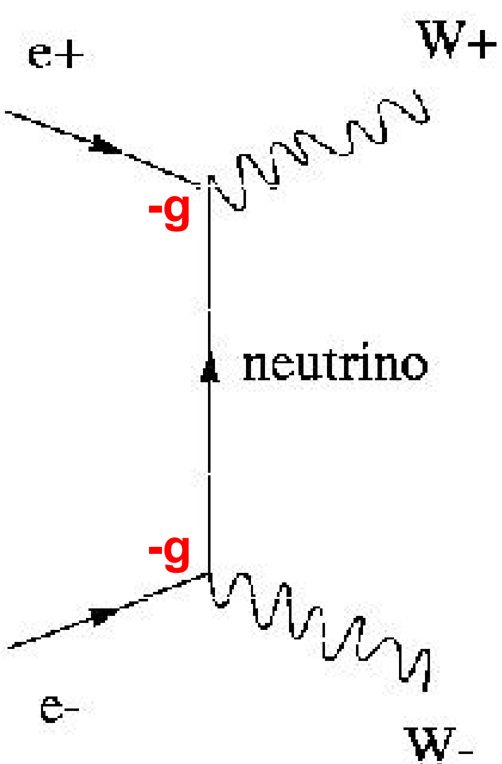
Intuitive explaination: mass makes interaction short-range (remember Yukawa theory), and therefore the field cannot compensate local phase transformations in the entire space, but only „nearby“.

Gauge theory of the Weak force?

- Gauge theories imply mass=0 for the force carriers; we demonstrated it for U(1) but the result is general
- At least two weak force carriers exist, both charged (W^+ and W^-) because weak decays change the charge:
  - $n \to p e \bar{\nu}$ interpreted as $n \to p W^{-(*)}$ plus $W^{-(*)} \to e \bar{\nu}$
- The "weakness" of the weak interaction suggests that the range is short, which can come in a natural way if the force carriers are quite massive (Yukawa theory)
- So, weak interaction seemed not to be explainable with a gauge theory, at first

What's the problem?

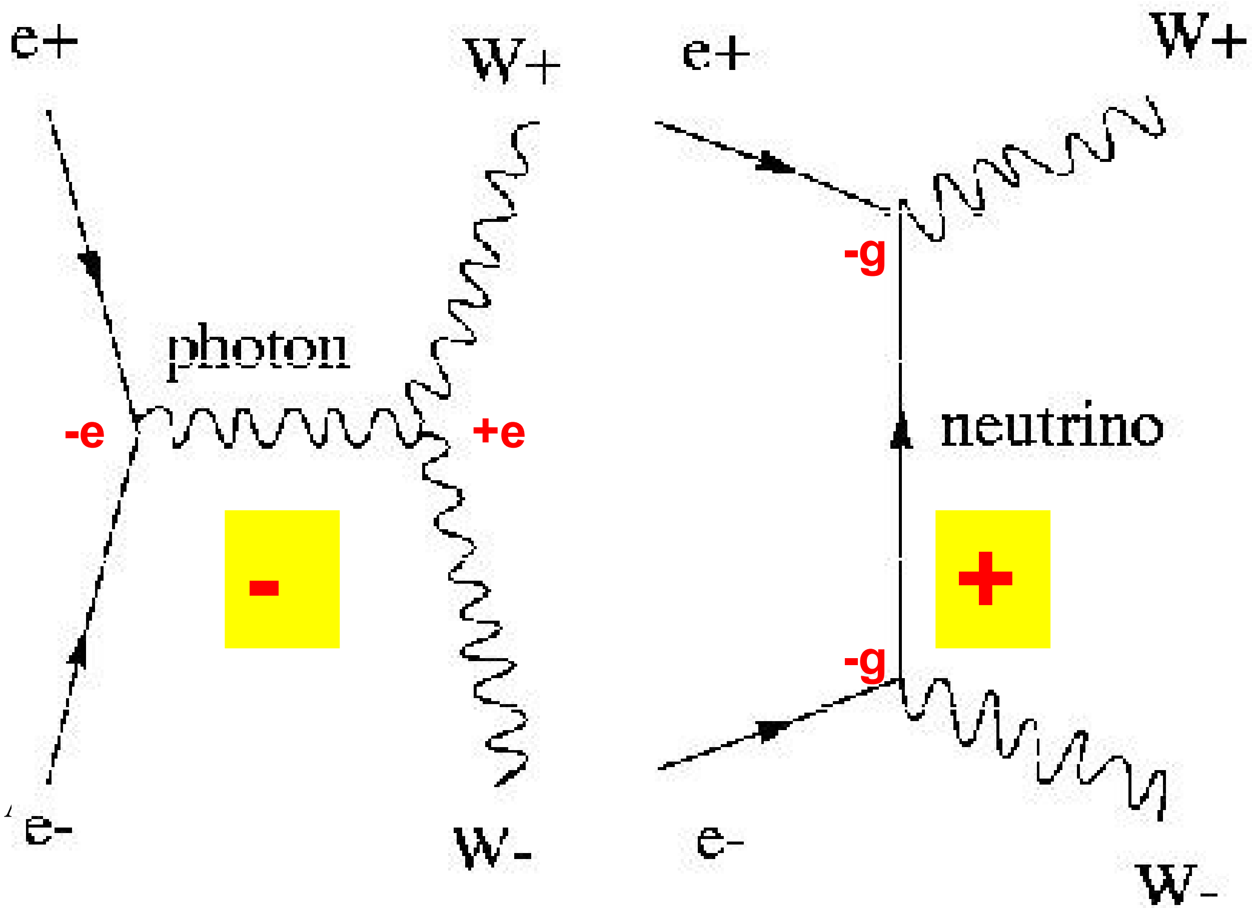
- A gauge theory is desirable because its symmetry provides cancellations of terms that would otherwise diverge
- A non-gauge quantum field theory will always contain somewhere some divergence that you cannot cancel



Example: if you calculate the probability that this process occurs in an e^+e^- collision, you get that above some energy the probability becomes >100%, i.e. $\sigma(e^+e^-\rightarrow W^+W^-) > \sigma(e^+e^-\rightarrow \text{anyth.})$!

Electro-Weak Unification?

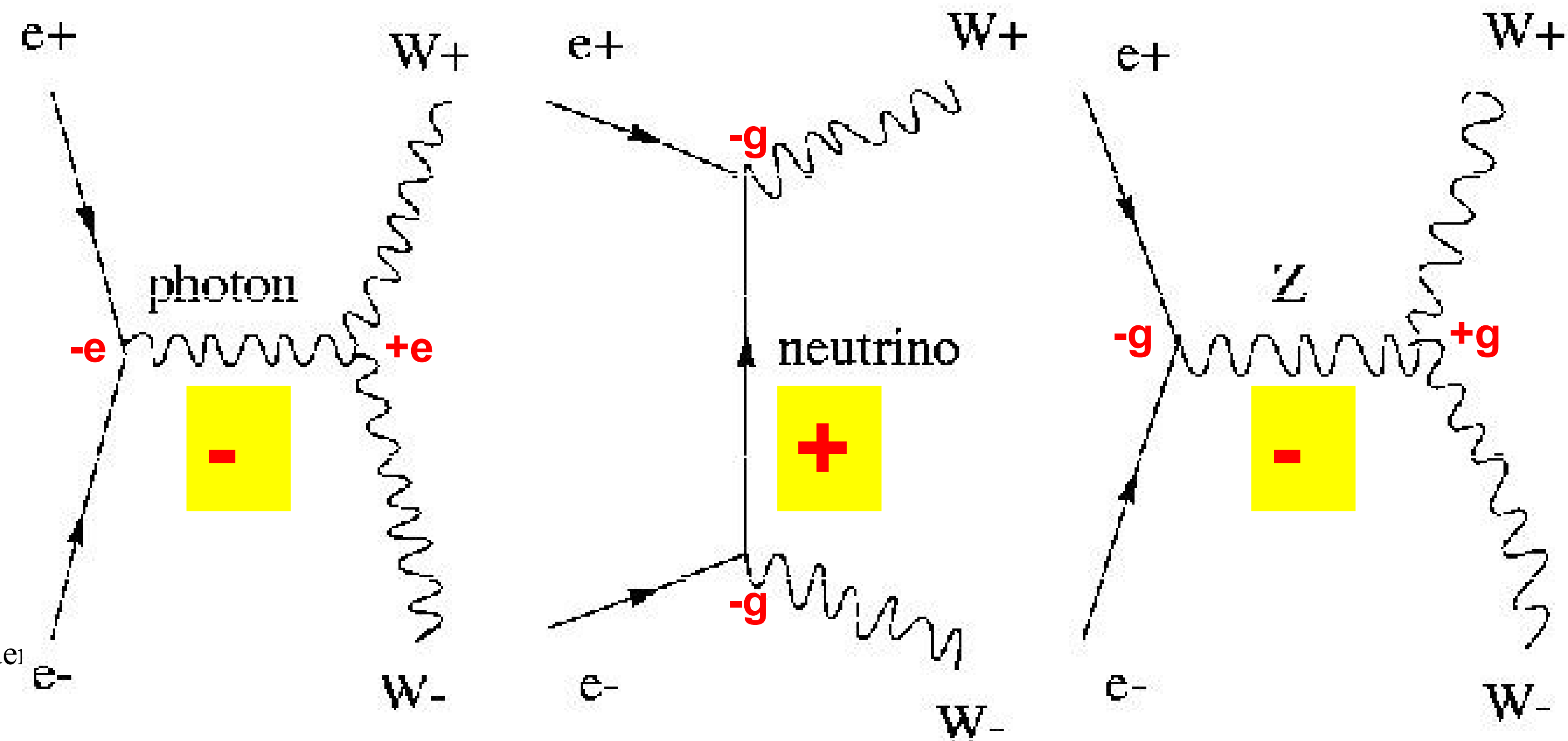
- A suggestive coincidence: if you assume g~e, summing the amplitudes of these two diagrams gives a cancellation up to some higher energy:



This delays the problem to higher energy, but does not solve it...

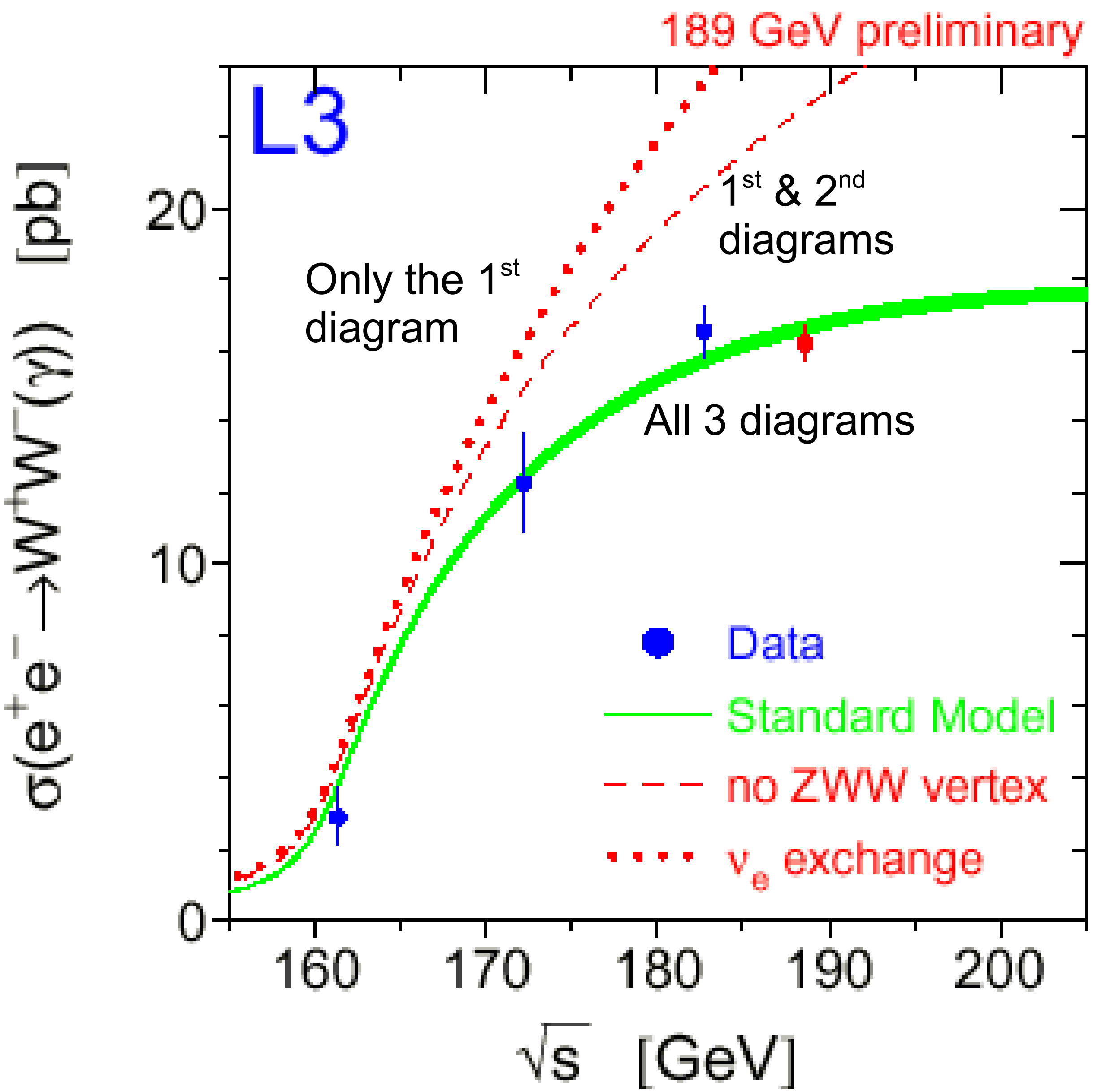
Electro-Weak Unification?

- Hypothesis by Glashow, Salam, Weinberg (1967): a new process where a new boson is present in the intermediate state (so that the sign of the amplitude is the same as for the diagram with the photon)



Experimental check

(many years later)



Towards Electro-Weak Unification

- If g~e, and if γ, W and Z masses are roughly equal (Electro-Weak unification), then there is a cancellation
- But of course these masses are not similar at all
  - photon is massless
  - W is heavy; if not, weak force would have long range
  - Z is heavy too, otherwise we would have observed it since long time, exactly as the photon
- The crucial idea:
  - the unified theory has a gauge symmetry (hence massless "native" vector bosons), that we don't see because it is broken by interaction with a scalar field (ϕ) that has a VEV$\neq$0

Weak isospin: SU(2)

Experimental evidence that left-handed fermions (and right-handed anti-fermions) are arranged in doublets, and that when interacting through a W boson we need to treat each doublet as a single object.

$$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ d_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

New quantum number: weak isospin. Because the basic object is a doublet, its gauge group is SU(2). Same algebra as normal spin.
I and I_3 are conserved; interaction with a $W^\pm$ makes a change of ±1 on the value of I_3, which is the mathematical way to say that *up↔down*.

Experimentally we also know that right-handed fermions do not interact with W's. So they are singlets of isospin ($I = I_3 = 0$).
The EM and strong forces do not make distinction by I or I_3.

SU(2)

This gauge group has three generators (Pauli matrices):

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In a gauge interaction, for each generator we have a vector boson; for SU(2) the three generators map into vector bosons W_1, W_2, W_3.

The combination that makes the ±1 change in I_3 is $W^{\pm} = ½(W_1 \pm iW_2)$.

The W_3 stays neutral, like a photon.

But W_3 cannot be the photon... *Q: why?*

SU(2) gauge invariance

Remember: ψ is now a doublet of spinors

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \Psi' = \left(I + i\frac{g}{2}\boldsymbol{\omega}(x) \cdot \boldsymbol{\tau} \right) \Psi$$

Note that for simplicity (without loss of generality) we are considering infinitesimal gauge transformation and we are using the definition of exponential for matrices:

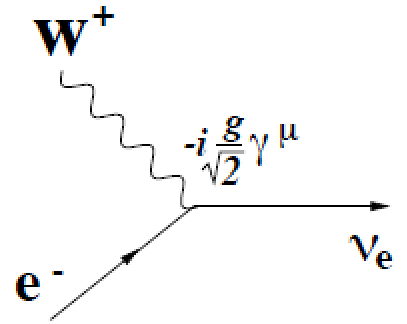
$$\begin{aligned} e^{A} &= \lim_{N\to\infty} \left(I + \frac{A}{N} \right)^{N} \\ e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}} &= \lim_{\epsilon\to 0} \left(I + i\,\epsilon\,\boldsymbol{\alpha}\cdot\boldsymbol{\tau} \right)^{1/\epsilon} \end{aligned}$$

SU(2) gauge invariance and charged weak interaction

$$\begin{aligned} D^\mu &= \partial^\mu + i\frac{g}{2}\boldsymbol{W}^\mu \cdot \boldsymbol{\tau} \\ \mathcal{L}_\mathrm{D} &= i\overline{\Psi}\gamma^\mu D_\mu \Psi \end{aligned}$$

$$\Psi_e = \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}, \quad \overline{\Psi}_e = (\bar{\psi}_{\nu_e}, \bar{\psi}_e), \quad \boldsymbol{W}^\mu \cdot \boldsymbol{\tau} = \begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix}$$

$$\frac{g}{\sqrt{2}}\bar{\psi}_{\nu_e}\gamma^\mu W_\mu^+ \psi_e \Rightarrow$$



Parity violation

We know experimentally (*Q: how?*) that $W^{\pm}$ bosons only interact with left-handed fermions and right-handed anti-fermions, which in spinor algebra are expressed as:

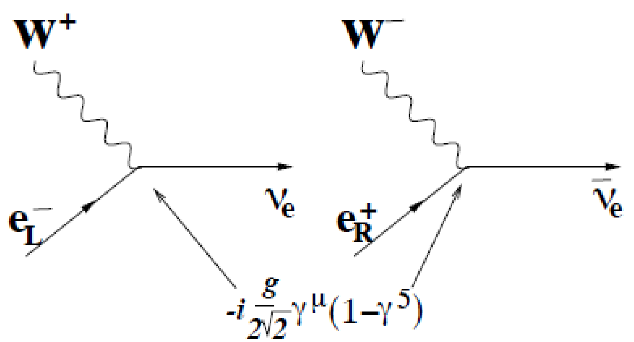
$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi \qquad (\bar{\psi})_R = \overline{\psi_L} = \frac{1}{2}\bar{\psi}(1+\gamma^5)$$

So let's re-write the previous formulas with ψ_L instead of ψ:

$$\begin{aligned}
\overline{\Psi}_L\gamma^\mu \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}\Psi_L &= \tfrac{1}{4}\overline{\Psi}(1+\gamma^5)\gamma^\mu \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}(1-\gamma^5)\Psi \\
&= \tfrac{1}{4}\overline{\Psi}\gamma^\mu(1-\gamma^5)^2 \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}\Psi \\
&= \tfrac{1}{2}\overline{\Psi}\gamma^\mu(1-\gamma^5) \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}\Psi
\end{aligned}$$

We used the property: $(1-\gamma^5)^2 = 1 + (\gamma^5)^2 - 2\gamma^5 = 2(1-\gamma^5)$

Parity violation



$$\begin{aligned}
\overline{\Psi}_L \gamma^\mu \boldsymbol{W}_\mu \cdot \boldsymbol{\tau} \Psi_L &= \tfrac{1}{4}\overline{\Psi}(1+\gamma^5)\gamma^\mu \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}(1-\gamma^5)\Psi \\
&= \tfrac{1}{4}\overline{\Psi}\gamma^\mu(1-\gamma^5)^2 \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}\Psi \\
&= \tfrac{1}{2}\overline{\Psi}\gamma^\mu(1-\gamma^5) \boldsymbol{W}_\mu \cdot \boldsymbol{\tau}\Psi
\end{aligned}$$

Mass of the fermions

With some algebra, we can rewrite the mass term of the fermions as function of the right-handed and left-handed spinors:

$$
\begin{aligned}
m\bar{\psi}\psi &= \tfrac{1}{4}m\bar{\psi}(1-\gamma^5)(1-\gamma^5)\psi + \tfrac{1}{4}m\bar{\psi}(1+\gamma^5)(1+\gamma^5)\psi \\
&= m\overline{\psi_R}\psi_L + m\overline{\psi_L}\psi_R
\end{aligned}
$$

Written this way, we realize at a glance that we have a problem: this term is not gauge-invariant! In fact, ψ_L transforms under SU(2) rotations, while ψ_R does not (because it is a singlet), so there cannot be any cancellation.

Fermions, like the vector bosons, are imposed by gauge invariance to be massless (in contradiction with experimental evidence.)

We will see later how we solve this problem.

Mass of the fermions

By the way, please don't see these operations as *pure mathematics*, but always as a mean to enhance your intuition of physics!

$$\begin{aligned} m\bar{\psi}\psi &= \tfrac{1}{4}m\bar{\psi}(1-\gamma^5)(1-\gamma^5)\psi + \tfrac{1}{4}m\bar{\psi}(1+\gamma^5)(1+\gamma^5)\psi \\ &= m\overline{\psi_R}\psi_L + m\overline{\psi_L}\psi_R \end{aligned}$$

Physics intuition of these two lagrangian terms: the first one turns a left-handed particle into a right-handed one; the second does the opposite.

So you can think of the mass term as an operator that connects states of opposite chirality. The larger the mass, the larger the L↔R connection.

A massless particle means a particle flying at the speed of light; such a particle is always in a definite chirality state.

You studied that when calculating the branching ratios of $\pi\to\mu\nu$ and $\pi\to e\nu$

SU(2)⊗U(1)

Remember: we would like to unify the Weak force, which seems more or less ok to be described by the SU(2) group, and the EM force, which is definitely described by the U(1) group.

It would be great to use SU(2) to explain everything, but the W^3 field of the SU(2) gauge cannot be interpreted as the EM field, as we saw before.

So the system must be simultaneously gauge invariant by SU(2) and U(1). Trivially stitching together $SU_{weak}(2)$ and $U_{EM}(1)$ is appealing, matematically.

$$\Psi \to \Psi' = \exp\left(ig\boldsymbol{\omega}\cdot\boldsymbol{I} + ig'\omega_0 Y\right)\Psi$$

$$D_\mu = \partial_\mu + ig\boldsymbol{W}_\mu\cdot\boldsymbol{I} + ig'B_\mu Y$$

I: isospin operator (matrix)
Y: some scalar operator; in the EM case it would be the electric charge operator

(Note: no fundamental reason for g and g' to have the same value)

Electro-Weak Unification

Problem: the W^3 field must behave like the other W fields, i.e. only couple with left-handed fermions / right-handed anti-fermions. But experimentally we know that the *weak neutral currents* behave in an intermediate way, i.e., they violate parity but not maximally.

Solution: the native fields obey the full symmetry, and this symmetry is a SU(2)⊗U(1) gauge, but the physical fields (i.e., after symmetry breaking) are a rotation of them, which leave $U_{EM}(1)$ as residual symmetry.

$$W_3^\mu = \cos\theta_w Z^\mu + \sin\theta_w A^\mu$$

$$B^\mu = -\sin\theta_w Z^\mu + \cos\theta_w A^\mu$$

$$\Rightarrow \qquad g \sin\theta_w = g' \cos\theta_w = e$$

(Note: θ_w is a fundamental parameter of the theory)

Let's make a simpler example ("Higgs model", 1964)

To show how a VEV can allow a massive vector boson in a gauge theory, consider a simple theory with only ϕ and A, and U(1) gauge symmetry:

$$L = \frac{1}{2}(D_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4$$ (+ kinetic energy of A)

$$\phi(x) \rightarrow e^{+ig\theta(x)} \phi(x)$$

$$\hat{\phi}(x) \rightarrow e^{-ig\theta(x)} \hat{\phi}(x)$$

$$a^2 \equiv \hat{a}\, a$$

$$D_\mu = \partial_\mu - i g A_\mu$$ with rule $$A_\mu \rightarrow A_\mu + \partial_\mu \theta$$

Let's make a simpler example ("Higgs model", 1964)

As before, let's now write $\phi(x) = v + h(x)$. At home, please do the exercise of this change of variable in the lagrangian of the previous page.

You will obtain, among others, this new term:

$$L_{mass\,,A} = \frac{1}{2}\left(D_\mu v\right)^2 = \frac{1}{2}\left(g\,v\right)^2 A^\mu A_\mu$$

Through its interaction with field ϕ, and via spontaneous symmetry breaking, the force field A has acquired an effective mass m_A=gv

The analoguous exercise in the SU(2)⊗U(1) case is more tedious, but conceptually identical. The W boson mass is proportional to the strength of the Higgs-W coupling and to the VEV, just like A in this toy model.

Mass of the fermions

The same Brout-Englert-Higgs mechanism can also solve, in passing, the problem of the mass of the fermions. It is just sufficient to make the assumption that the field ϕ and the fermions interact.

The simplest interaction term is the so called Yukawa term:

$$L_{Yukawa} = y \bar{\psi} \psi \phi$$

Here the constant y is the strength of the interaction.

If there is spontaneous symmetry breaking, as usual, $\phi(x) = v + h(x)$

$$L_{Yukawa} = y v \bar{\psi} \psi + y \bar{\psi} \psi h$$

(Note: when doing the real calculation in SU(2) there is a $\sqrt{2}$ factor)

Mass of the fermions

$$L_{Yukawa} = \frac{yv}{\sqrt{2}} \bar{\psi} \psi + \frac{y}{\sqrt{2}} \bar{\psi} \psi h$$

The first term is a mass term! The mass of the fermions can be accommodated by the Brout-Englert-Higgs mechanism.

Intuitive explanation: the ϕ field slows down the fermion by continuously scattering with it. The ϕ field is "dense" in the vacuum because the ground state is not at $\langle\phi\rangle=0$ but at $\langle\phi\rangle=v$ $\Rightarrow$ large v means large mass.

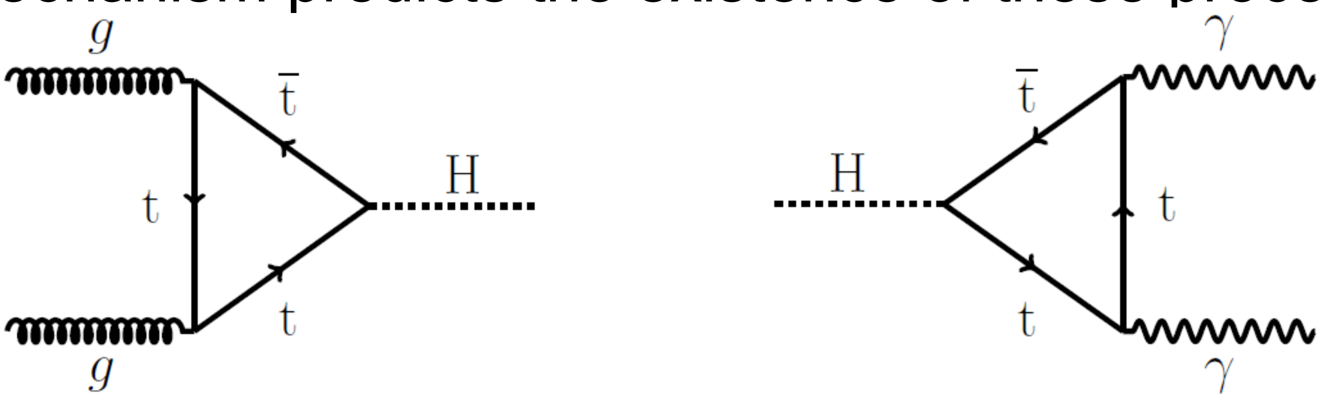
The stronger the Yukawa interaction, the more often the fermion is scattered and therefore slowed down $\Rightarrow$ large y means large mass.

Problem: there is no reason why y should be the same for all fermions; and in fact, each fermion has a different mass, hence a different y. And the SM has nothing to say about the values of these constants.

Effective couplings to m=0 particles

$$L_{Yukawa} = \frac{yv}{\sqrt{2}}\bar{\psi}\psi + \frac{y}{\sqrt{2}}\bar{\psi}\psi h$$

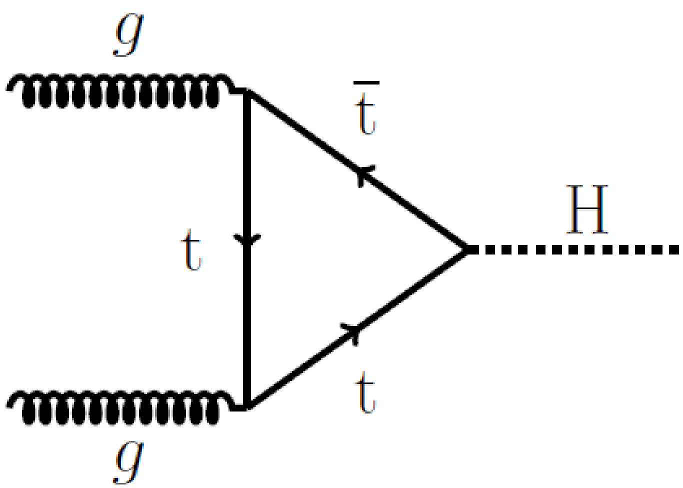
If the mass of the fermions comes from the BEH mechanism, the same mechanism predicts the existence of these processes:

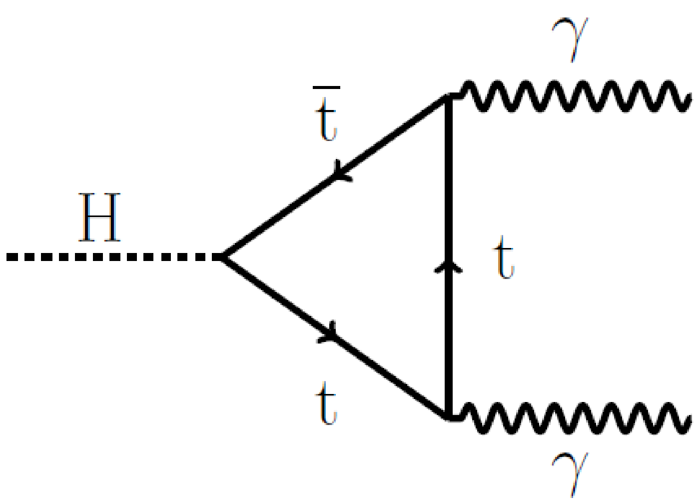


Remember: the Higgs boson has no coupling (by construction!) with massless particles. But these loops create effective couplings.

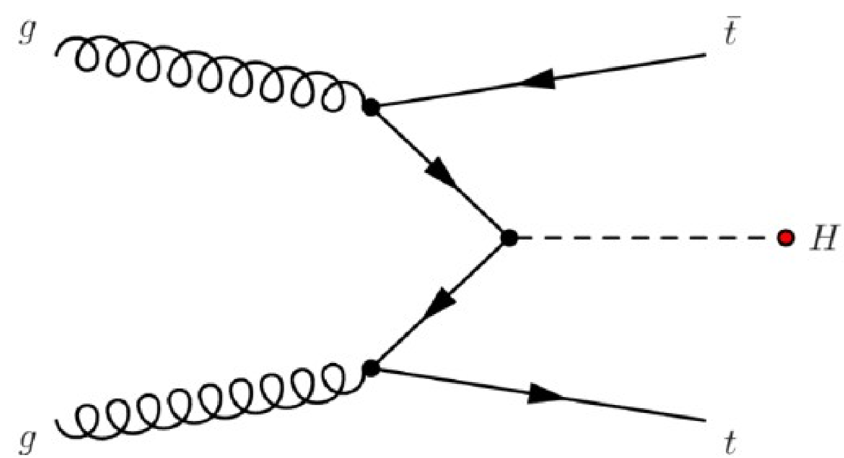
Q: why do we usually plot only the top quark in these loops?

Measuring $y_{(t)}$ as test of the SM





Indirect: through loops



Direct: at tree level

By measuring all three, one can extract y_t and compare with $m_t\sqrt{2}/v$.

By comparing indirect vs direct, one checks whether there are other heavy particles running in the loops.

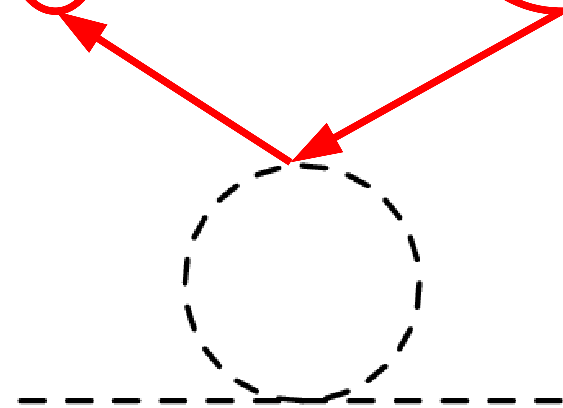
Take-home messages

- The Standard Model is built from a mix of theory considerations (e.g., renormalizability) and experimental constraints (e.g., parity violation, need to explain mass, etc.)
- It was a big conceptual progress, as it explains several disconnected phenomena with a small set of lagrangian terms
- However, several pieces look arbitrary, for example the values of the fundamental parameters are not explained (and some of them look "weird", e.g., the fermion mass hierarchy)
- General consensus: the SM is an incomplete theory, most probably the low-energy limit of the true theory

Questions?

To be kept in mind for later...

$$L = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \left(\lambda v h^3 + \frac{1}{4}\lambda h^4\right) + const.$$



This kind of loop diagrams affects the physical (renormalised) Higgs mass.

We will see at some point later that this sensitivity to loop corrections is the deep source of the Fine Tuning Problem, or Hierarchy Problem.