Astrophysics and Nuclear Astrophysics

(LPHY2263)

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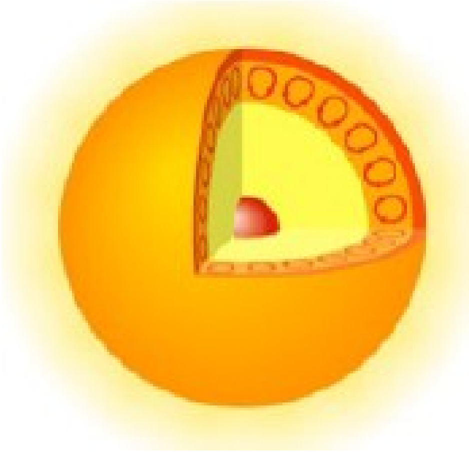
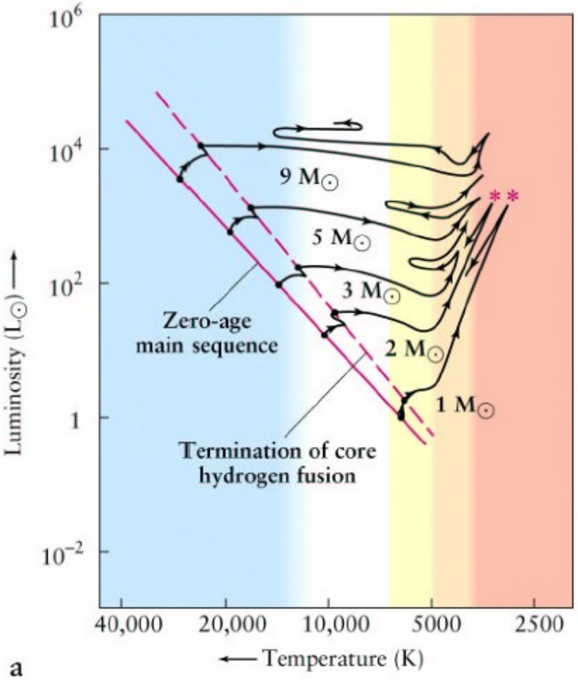
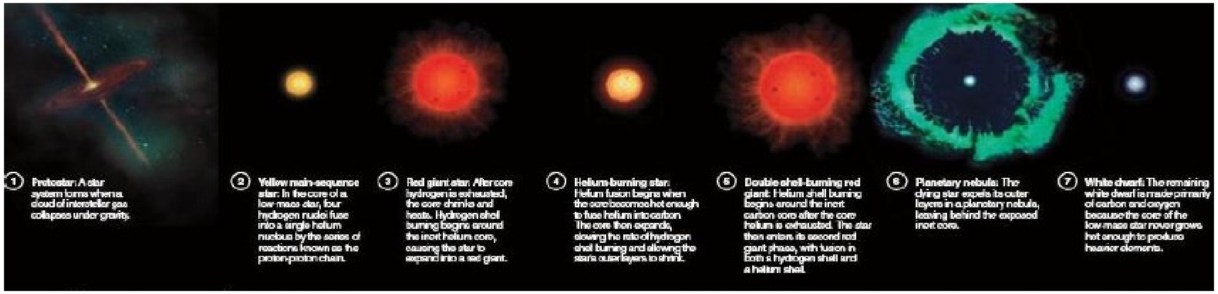
Chapter #4

- Energy transport in a star
  - Convection versus radiation
- Lifetime-mass relationship
- Death of a Sun-like star
  - Exiting from the Main Sequence
  - AGBs
  - White dwarfs







① Protostar: A star system forms when a cloud of interstellar gas collapses under gravity.

② Yellow main-sequence star: In the core of a low-mass star, four hydrogen nuclei fuse into a single helium nucleus by the series of reactions known as the proton-proton chain.

③ Red giant star: After core hydrogen is exhausted, the core shrinks and heats. Hydrogen shell burning begins around the inert helium core, causing the star to expand into a red giant.

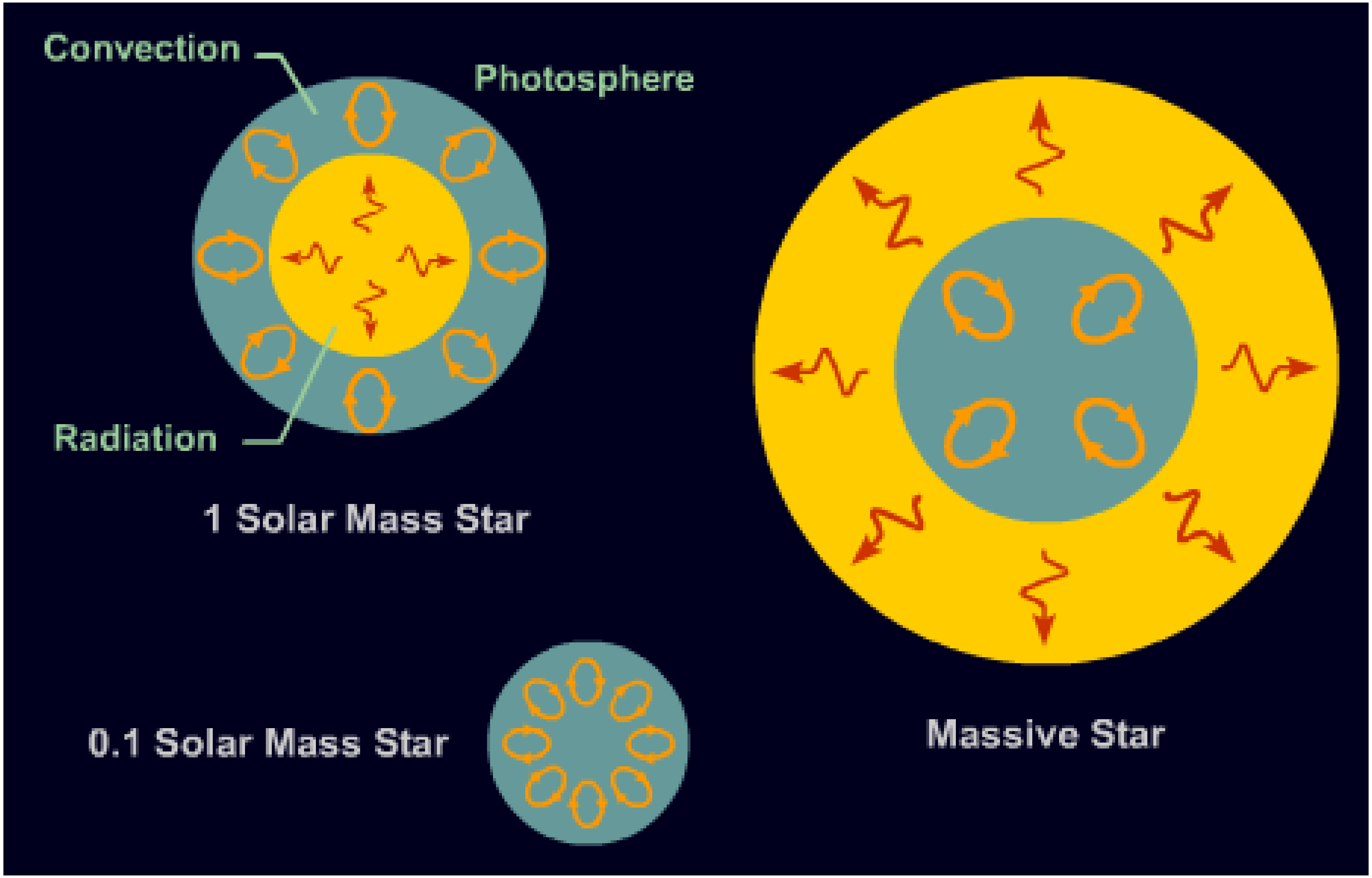
④ Helium-burning star: Helium fusion begins when the core becomes hot enough to fuse helium into carbon. The core then expands, slowing the rate of hydrogen shell burning and allowing the star's outer layers to shrink.

⑤ Double shell-burning red giant: Helium shell burning begins around the inert carbon core after the core helium is exhausted. The star then enters its second red giant phase, with fusion in both a hydrogen shell and a helium shell.

⑥ Planetary nebula: The dying star expels its outer layers in a planetary nebula, leaving behind the exposed inert core.

⑦ White dwarf: The remaining white dwarf is made primarily of carbon and oxygen because the core of the low-mass star never grows hot enough to produce heavier elements.

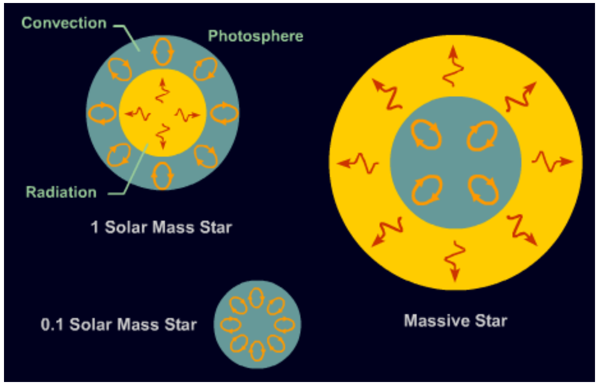
Convection and radiation



Why?

Main fusion mechanisms and internal structure

- Fusion reaction rates depend on T: $\varepsilon(PP) \sim T^4$, $\varepsilon(CNO) \sim T^{17}$
- Classification in upper/lower/red Main Sequence:
  - $M>1.2M_{sun}$:
    - $T_{core} > 18$ MK; most energy from CNO cycle
    - Structure: convective core, radiative envelope
  - $0.25M_{sun}<M<1.2M_{sun}$:
    - $T_{core} < 18$ MK; most energy from PP cycle
    - Structure: radiative core, convective envelope
  - $0.08M_{sun}<M<0.25M_{sun}$:
    - Most energy from PP cycle
    - Structure: fully convective
  - Below $0.08M_{sun}$, no fusion: "brown dwarf"



What is convection?

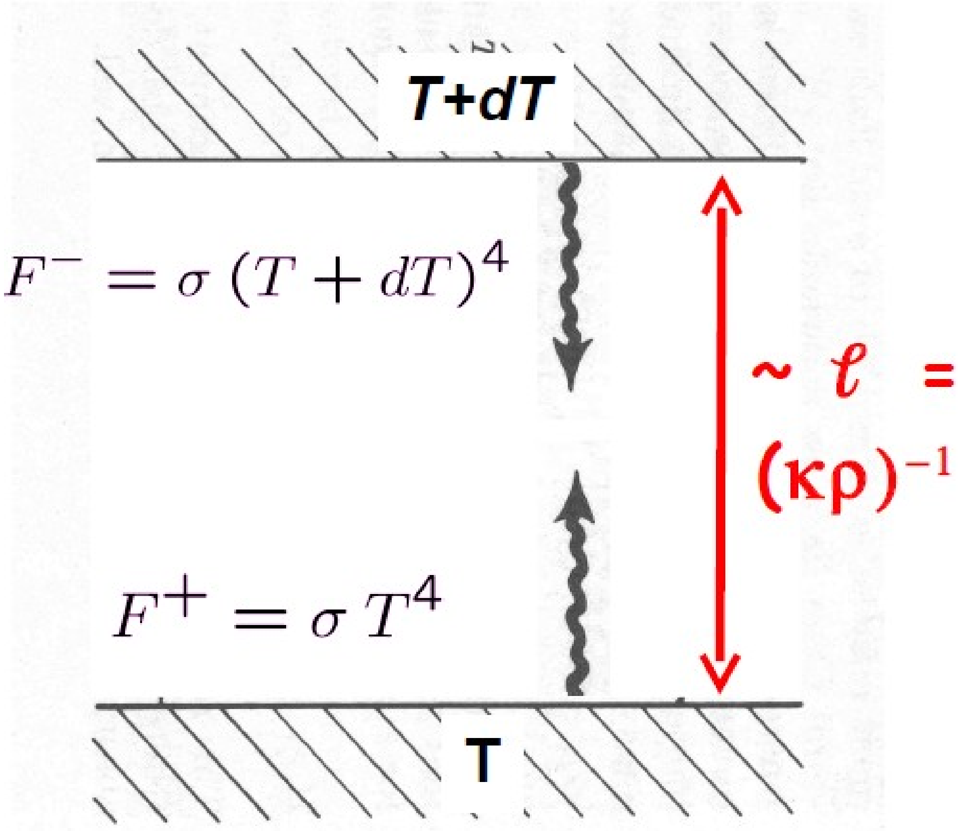
- It is an ***adiabatic*** movement of matter
  - Think about a bubble of liquid that for some reason has a different T than the surrounding liquid, therefore expands, and floats up
  - Adiabatic means that there is no energy exchange with surrounding; this can happen if the bubble rises *too fast to thermalize*
  - But in the following we will also assume that it is *slow enough* to reach pressure equilibrium, i.e., $v \ll v_{\text{sound}}$

When convection?
When radiation?

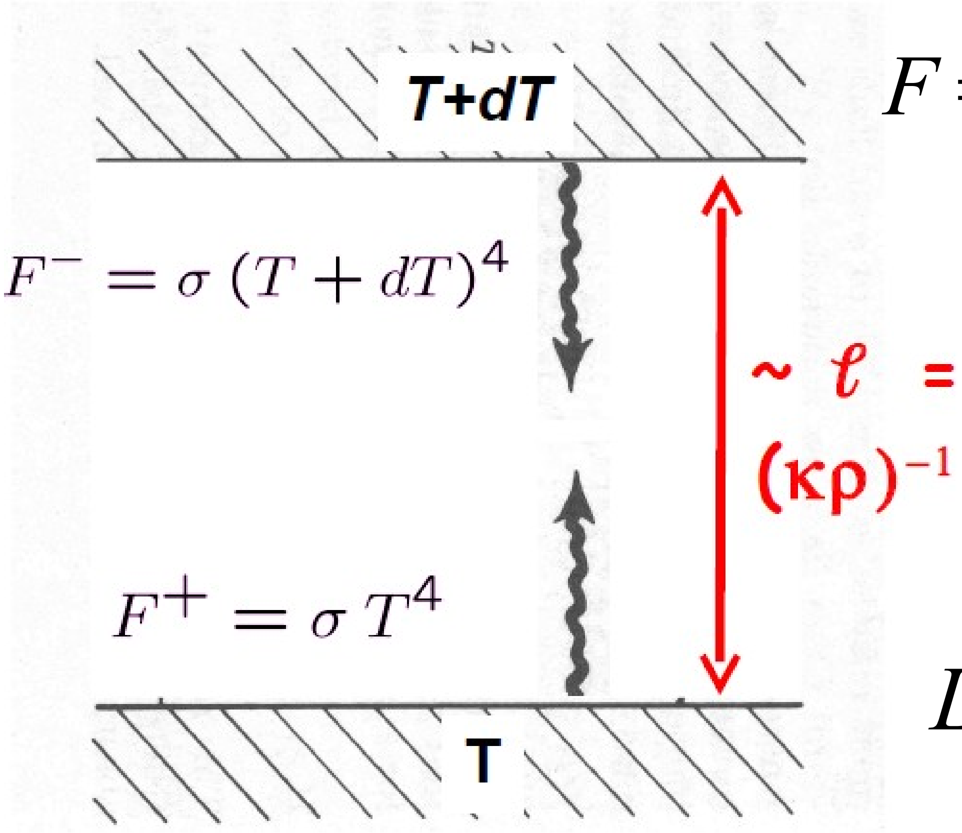
- Radiation dominates when it is a more efficient mechanism for energy transport than convection
  - The third famous energy transport mechanism, conduction, is negligible for a gas
- Calculation starts by estimating the temperature gradient dT/dr under the assumption that only radiation happens: $(dT/dr)_{\mathrm{rad}}$
- Then the Schwartzschild criterion tells you if the system is stable against convection or not

Temperature gradient by radiation

- The "free path" of a particle (here a photon) in a medium (here the star's gas) is the average distance that it manages to travel without interacting
- We can write $l_{free} = 1/(\kappa\rho)$, where κ is called "coefficient of opacity" (units: m^2/kg)
- Sun is very opaque: $l_{free} \sim 1$ cm
- Take two small volumes, separated by $O(l_{free})$
- As usual, assume a Black Body
  - Energy flux goes like T^4



Temperature gradient by radiation



$$F=\sigma\left(T+\delta T\right)^4-\sigma T^4\approx 4\sigma T^3\,\delta T$$

$$\frac{\delta T}{l}\approx-\frac{dT}{dr}$$

$$L=4\pi r^2 F=-\frac{16\pi r^2\sigma T^3}{\kappa\rho}\frac{dT}{dr}$$

Schwartzschild criterion (1)

- How large must be the temperature gradient (dT/dr) for convection to be the dominant phenomenon?
- Consider bubble in equilibrium with surrounding
- Both bubble and surrounding have ρ_1, T_1, P_1
- Imagine a small perturbation in temperature, while keeping pressure equilibrium
- $T_1' > T_1$ then requires $\rho_1' < \rho_1$
- Archimedes' principle pushes the bubble up
- It will reach a new place with ρ_2, T_2, P_2
- After adiabatic expansion, pressure equilibrium $P_* = P_2$, but ρ_* is not necessarily equal to ρ_2
- If $\rho_* < \rho_2$, bubble continues to raise: convection wins

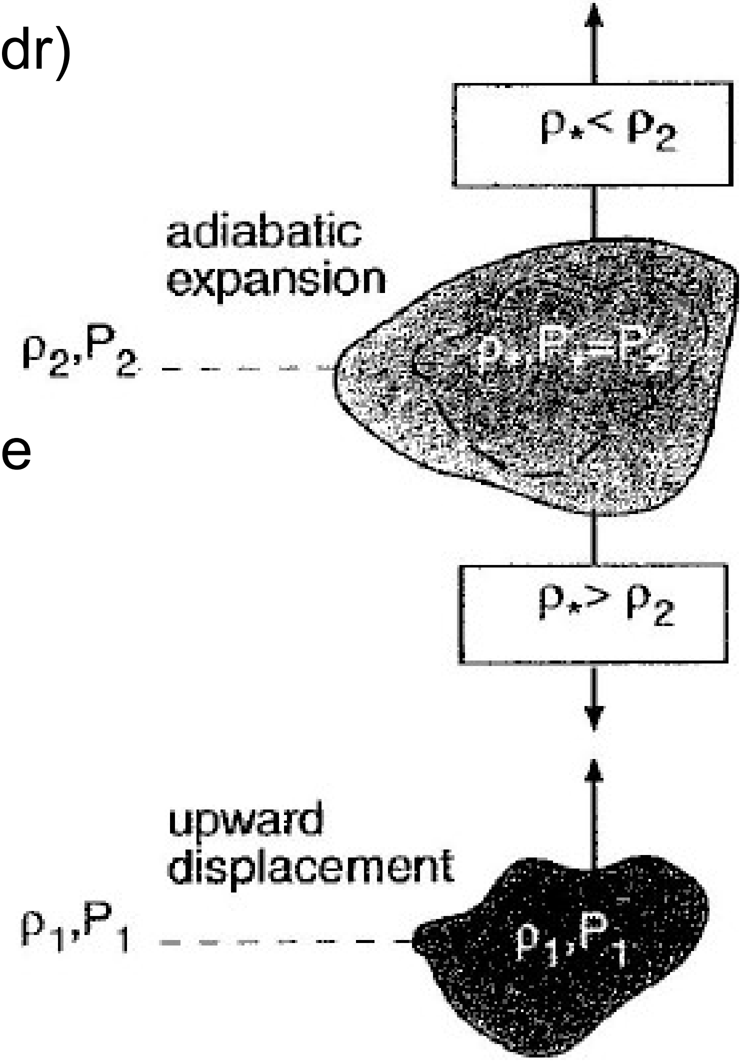


Image from http://www.star.ucl.ac.uk/~ljs/3c34/convection.pdf

Schwartzschild criterion (2)

- Change in internal density $\Delta\rho_i$, vs change in surrounding $\Delta\rho_s$
- Convection does <u>not</u> occur if:

$$\left|\frac{d\rho}{dr}\right|_i > \left|\frac{d\rho}{dr}\right|_s \quad\Longrightarrow\quad \left|\frac{dT}{dr}\right|_s < \left|\frac{dT}{dr}\right|_i \quad\Longrightarrow\quad \left|\frac{dT}{dr}\right|_{\mathrm{rad}} < \left|\frac{dT}{dr}\right|_{\mathrm{ad}}$$

- We assumed pressure equilibrium: P=const ⇒ ρT=const
- Surrounding is kept in equilibrium by radiation; *ad*: adiabatic
- To have stability against convection, radiative gradient must be smaller than adiabatic gradient
- Rewrite formula in a different way:

$$\frac{dT}{dr} = \frac{dT}{dP}\frac{dP}{dr} \quad\Longrightarrow\quad \left|\frac{dT}{dP}\right|_{\mathrm{rad}} < \left|\frac{dT}{dP}\right|_{\mathrm{ad}}$$

- Here we assumed pressure equilibrium with surrounding at all depths (implies $v \ll v_{\mathrm{sound}}$), hence dP/dr is the same

Schwartzschild criterion (3)

- For an adiabatic gas:

$$P \propto \rho^{\gamma} \qquad P \propto \rho T$$

- Adiabatic coefficient: $\gamma = C_p/C_v$ (specific heats C=dQ/dT at constant pressure or volume)
- For a perfect monoatomic gas which is completely ionized or completely neutral: $\gamma = 5/3$
- Combining these two equations:

$$T \propto \frac{P}{\rho} \propto \frac{\rho^{\gamma}}{\rho} \propto \rho^{\gamma-1} \propto \frac{P}{P^{1/\gamma}} \propto P^{\frac{\gamma-1}{\gamma}} \quad \Rightarrow \quad \frac{dT}{dP}\frac{P}{T} = \frac{\gamma-1}{\gamma} \quad \Rightarrow \quad \left|\frac{d\ln T}{d\ln P}\right|_{\mathrm{ad}} = \frac{dT}{dP}\frac{P}{T} = \frac{\gamma-1}{\gamma}$$

- Schwartschild's criterion for stability against convection:

$$\left|\frac{d\ln T}{d\ln P}\right|_{\mathrm{rad}} < \frac{\gamma-1}{\gamma} \qquad \text{or} \qquad \left|\frac{dT}{dr}\right|_{\mathrm{rad}} < \left|\left(\frac{\gamma-1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}\right|$$

When does convection dominate?

- Use the relationship that we had found for radiation:

$$L = 4\pi r^2 F = -\frac{16\pi r^2 \sigma T^3}{\kappa\rho}\frac{dT}{dr}$$

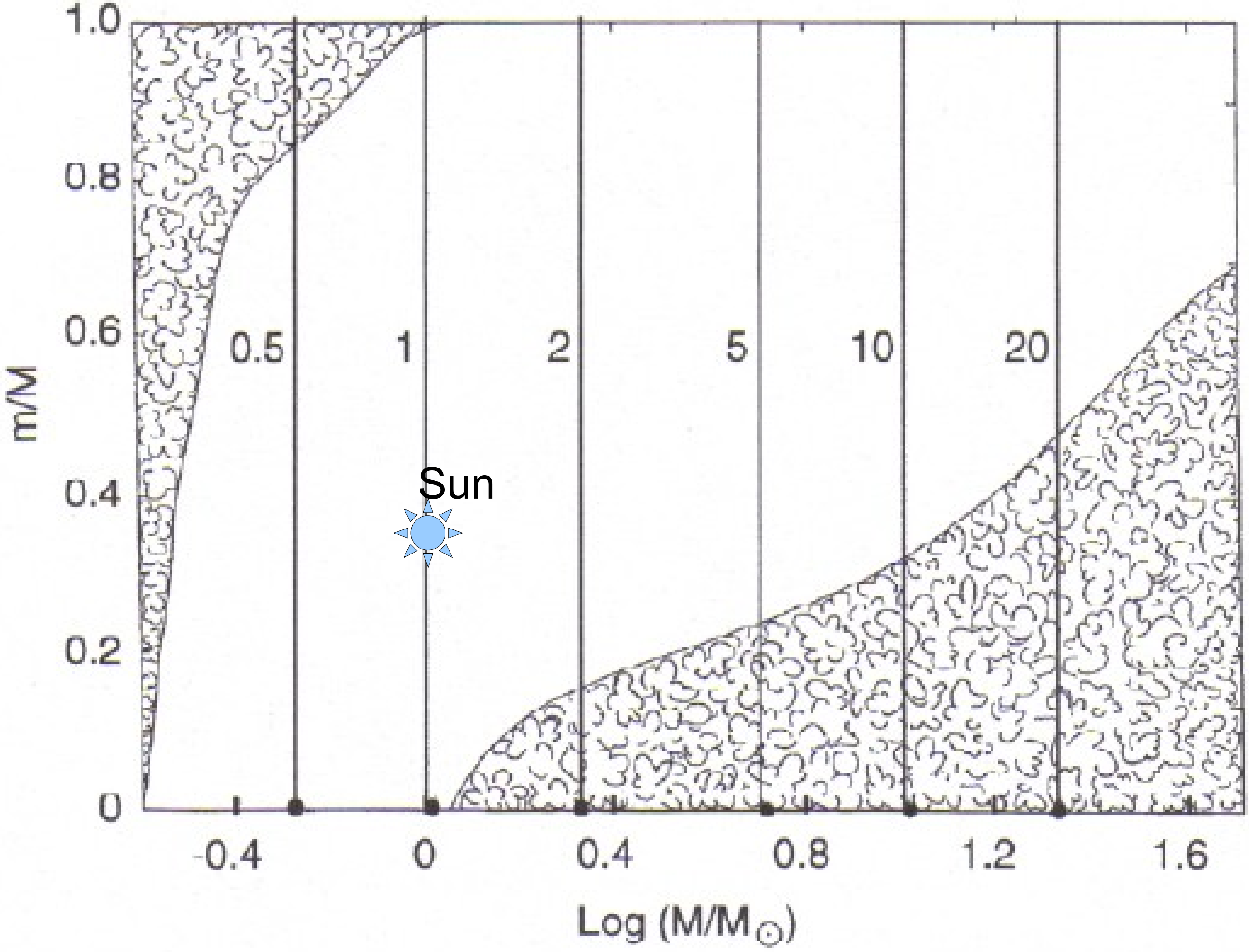
- At every inner radius r:

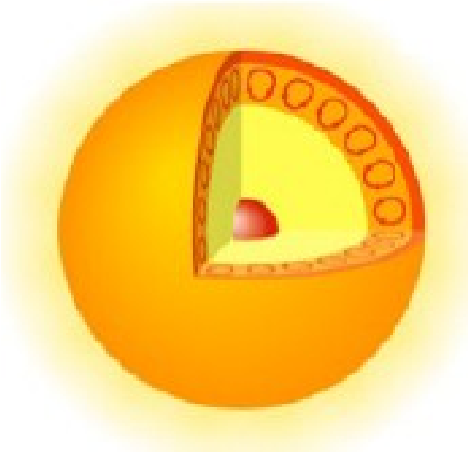
$$\left|\frac{dT(r)}{dr}\right|_{rad} \propto \frac{\kappa(T)\rho(r)L(r)}{r^2 T^3(r)}$$

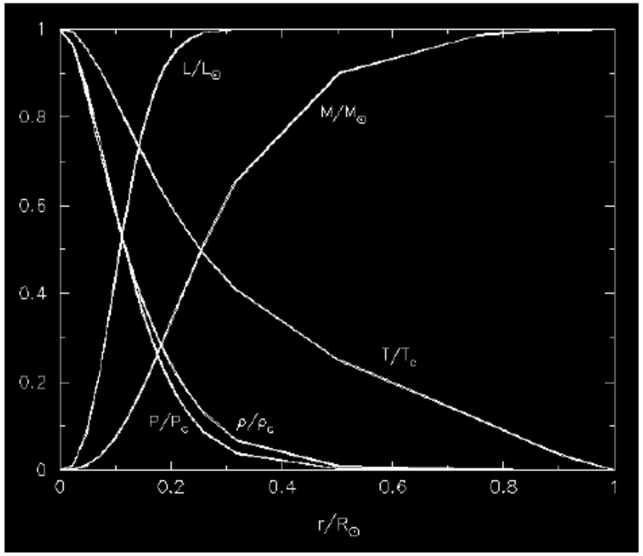
L(r) is <u>local</u> luminosity, etc.

- Convection dominates where:
  - Opacity (κ) is large; κ~1/T, and small stars are cold enough that their envelopes are opaque enough to cause convection
  - L(r) is large; for example, core of large stars is hot enough for CNO, which has very strong output

Convective zones

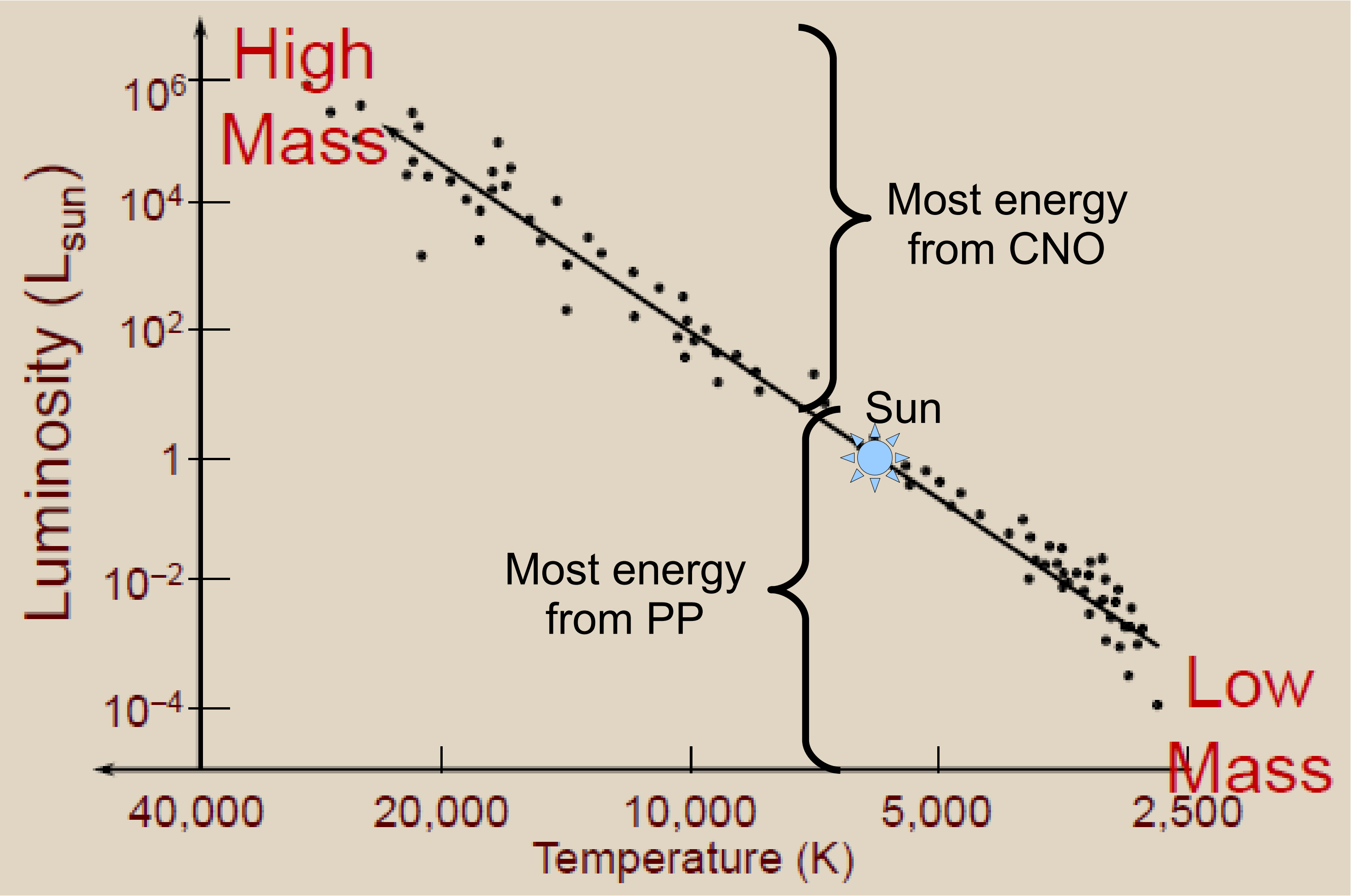






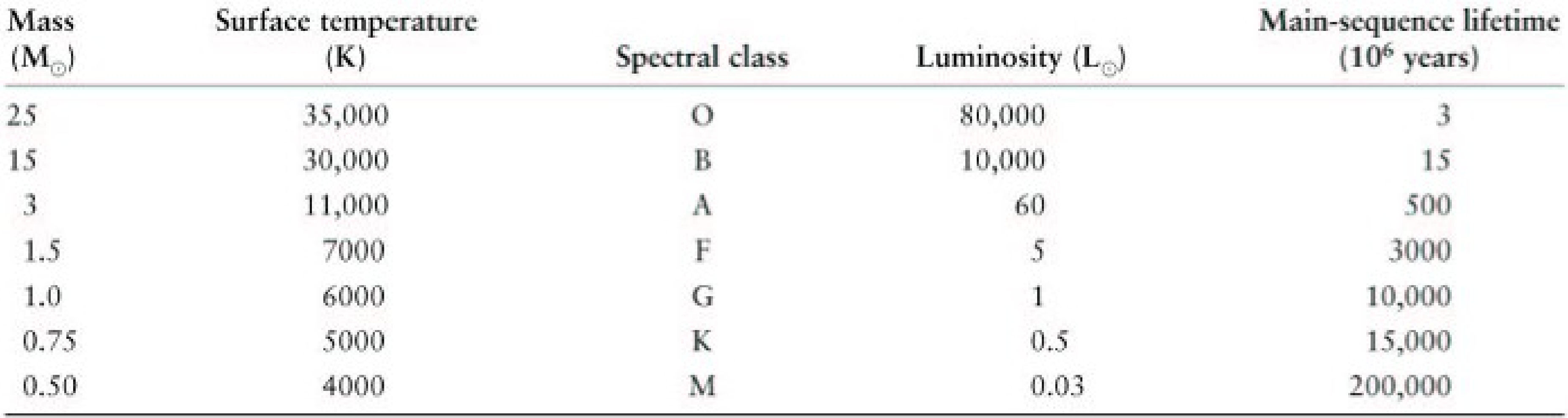
M: total mass; m: mass from 0 to r.
The convective zone in the Sun is large in r, but it is a low-density zone.

Main Sequence in the H-R diagram



Lifetime-mass relationship in the Main Sequence

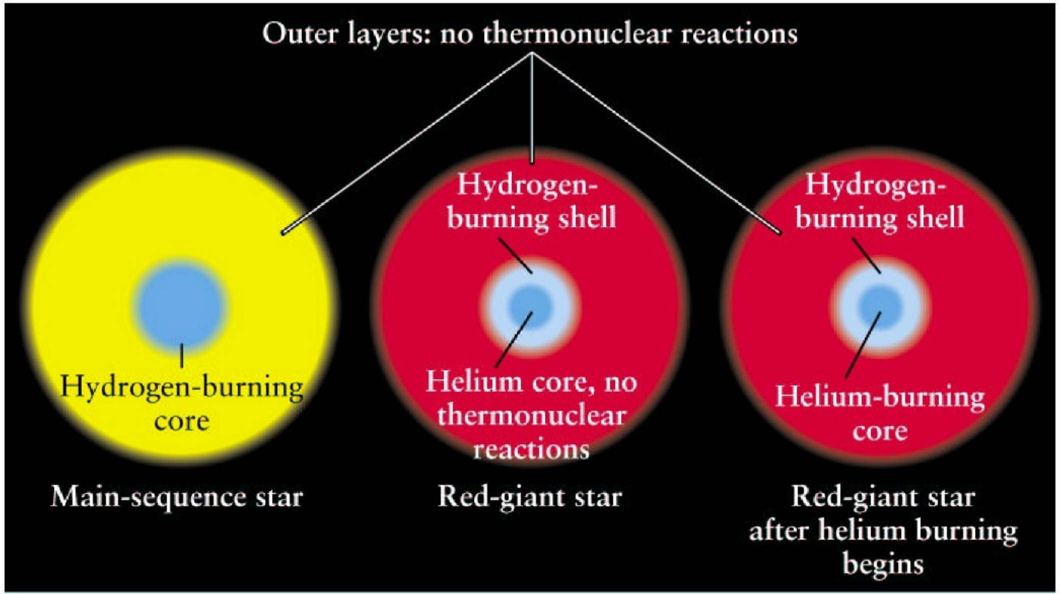
- A star can stay in the Main Sequence if:
  - It is in hydrostatic equilibrium
  - It burns H into He
- How long can it burn H into He? Depends on:
  - Amount of H available → mass of the star
  - How fast fusion occurs → luminosity
  - ⇒ Lifetime ~ mass/luminosity
- Remember: luminosity ~ mass$^{3.5}$
  - ⇒ Lifetime ~ mass$^{-2.5}$
  - Massive (hot) stars live less than light (cold) ones



Mass ($M_\odot$)	Surface temperature (K)	Spectral class	Luminosity ($L_\odot$)	Main-sequence lifetime (10^6 years)
25	35,000	O	80,000	3
15	30,000	B	10,000	15
3	11,000	A	60	500
1.5	7000	F	5	3000
1.0	6000	G	1	10,000
0.75	5000	K	0.5	15,000
0.50	4000	M	0.03	200,000

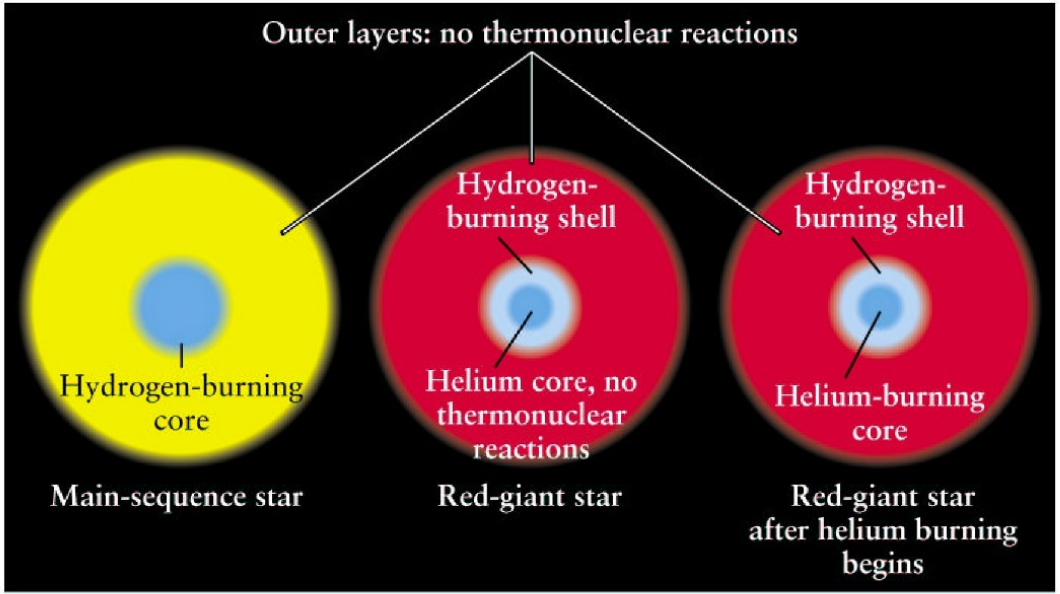
Evolution of a Sun-like star (1)

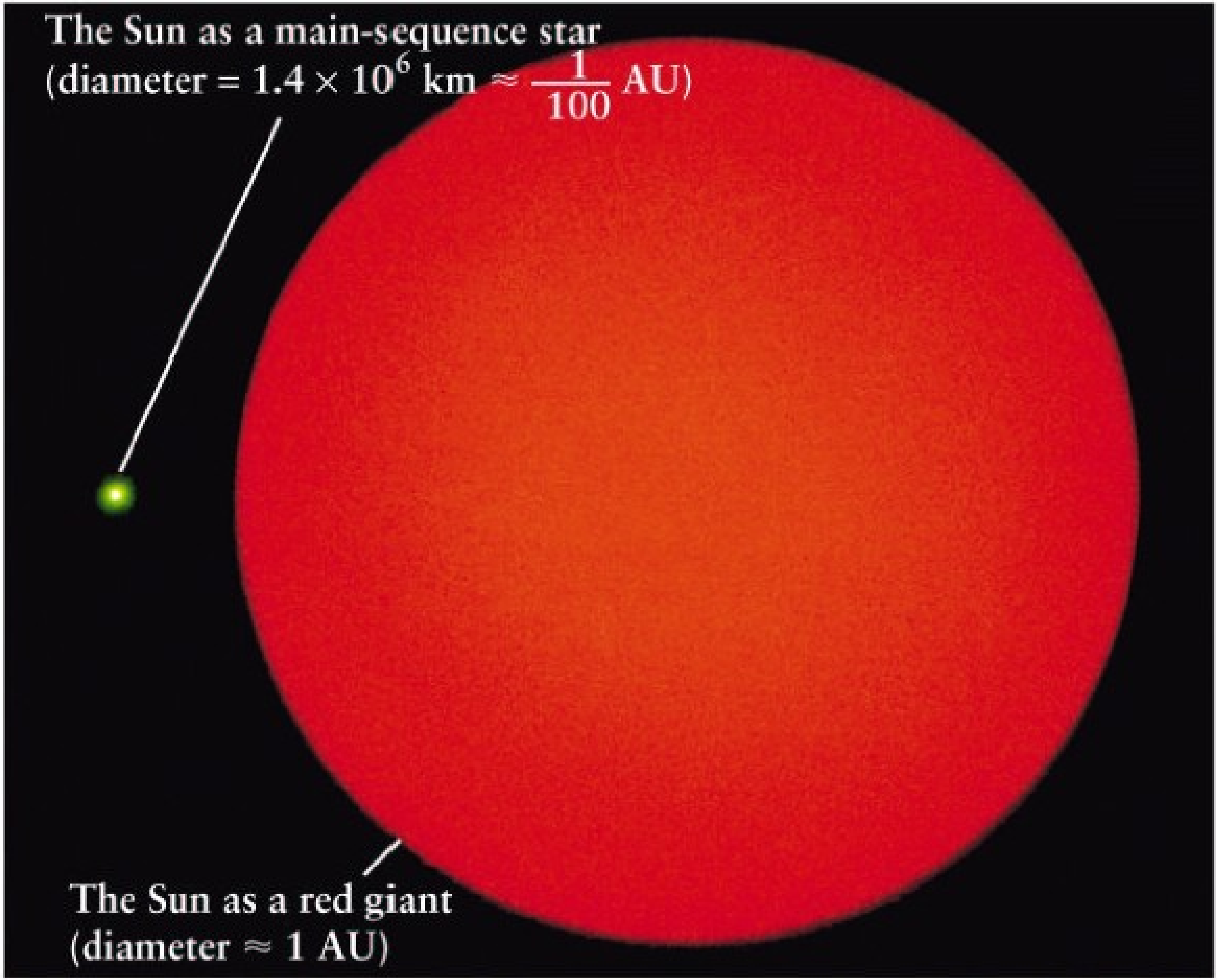
- Steady state in the Main Sequence:
  - H is slowly fused into He
  - T is too low for He fusion, He accumulates in the core
- At some point, not enough H in the core
  - Energy production decreases
  - He core starts to contract, liberating potential energy



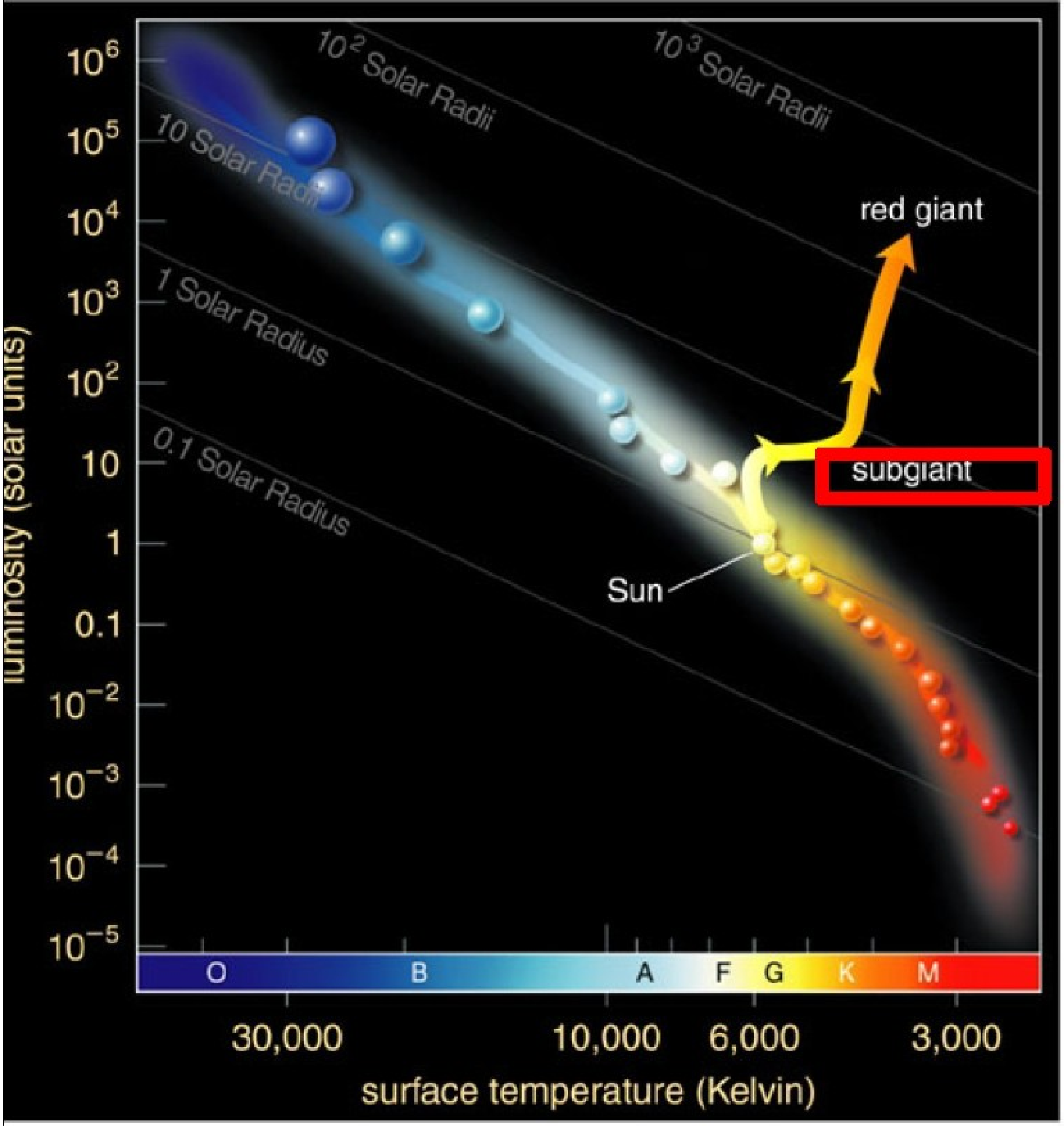
Evolution of a Sun-like star (2)

- Star becomes a Red Giant:
  - He contraction warms up the core
  - This also warms up the H shell around the core
  - H fusion rate increases in this shell
  - All intermediate layers warm up and expand → giant
  - Less surface temperature → red





Mercury, Venus and Earth are eaten by the Sun



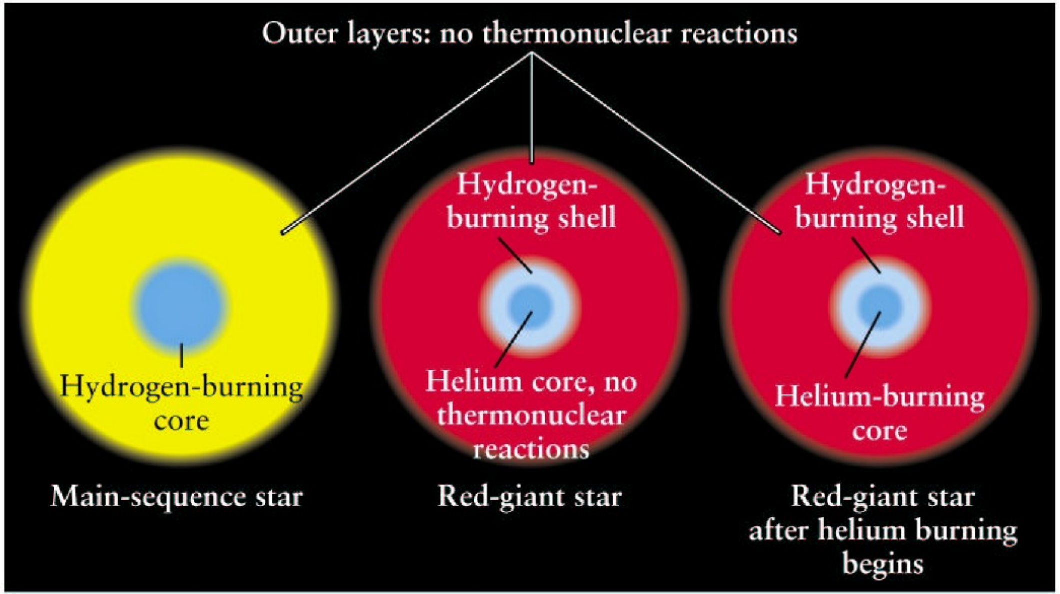
Red giants are cool but very luminous, thanks to large radius:

$$L = 4 \pi R^2 \sigma \; T_{eff}^4$$

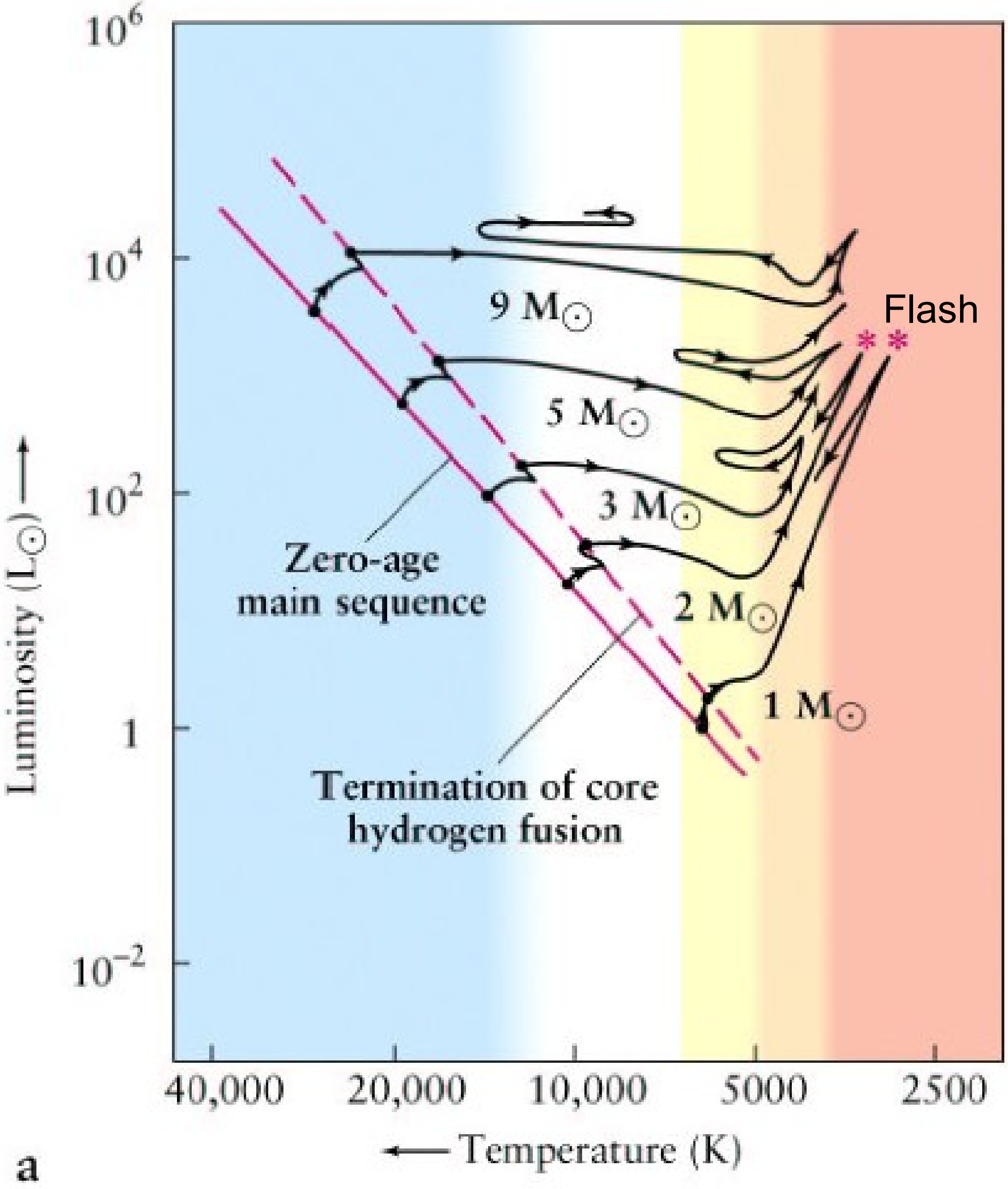
During sub-giant phase (~1-2 billion years), convection brings some of the fusion products (heavy elements) from the core to the surface

Evolution of a Sun-like star (3)

- Helium burning (~20% of star lifetime):
  - When core reaches 10^8 K, He fusion can start
    - *(Question: why is it larger than for H?)*
  - Dominant processes:
    - Triple-alpha: $^4He + ^4He \rightarrow ^8Be$ followed by $^8Be + ^4He \rightarrow ^{12}C + \gamma$
    - Production of Oxigen: $^{12}C + ^4He \rightarrow ^{16}O + \gamma$



Helium flash

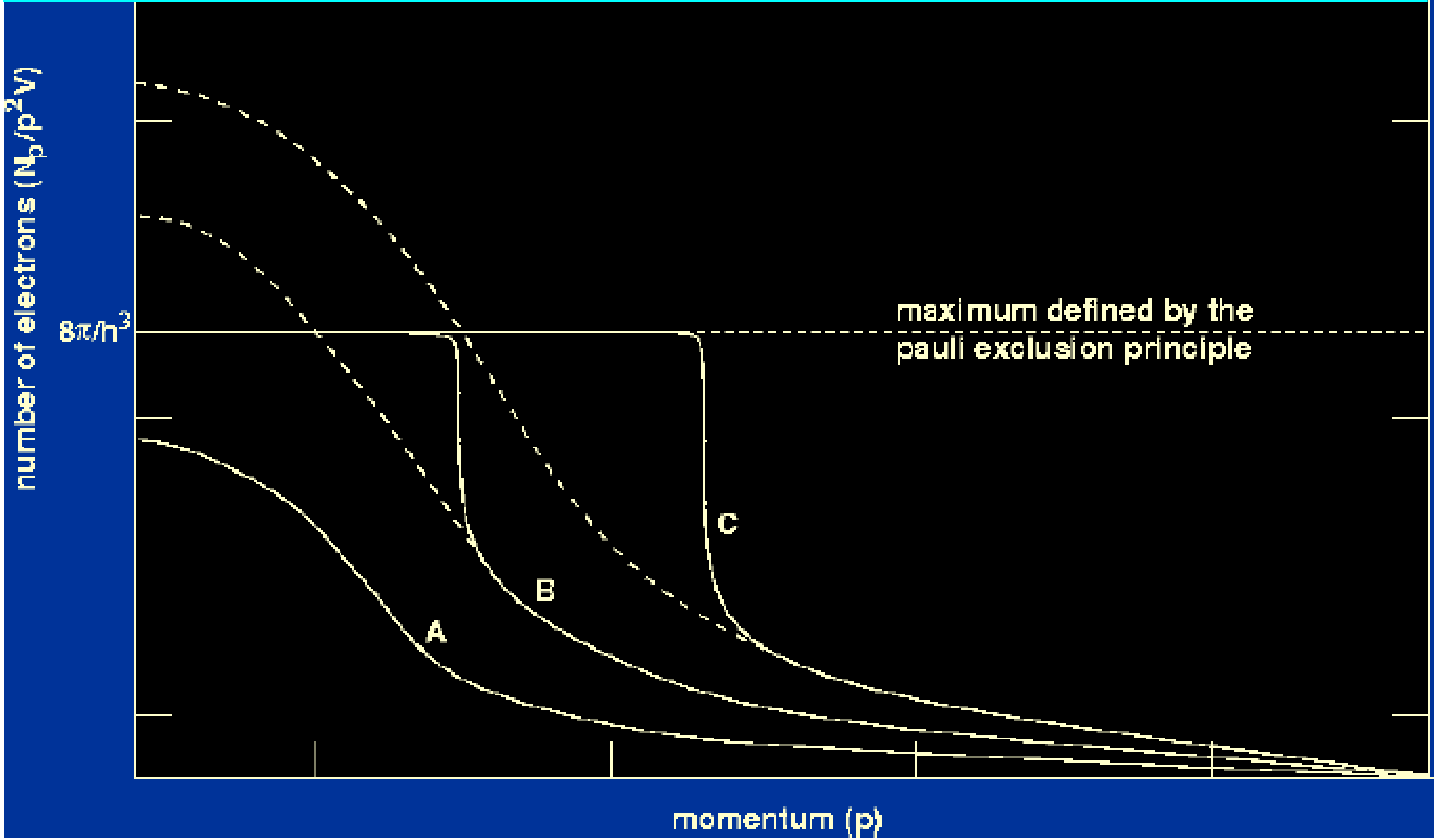


- Stars with $M/M_{Sun}>2.25$ have a gradual start of He fusion, which then stays stable for a while; hydrostatic equilibrium prevents runaway
- Instead, for $M/M_{Sun} < 2.25$ an interesting thing happens: for few seconds, local core luminosity becomes $\sim 10^{11} L_{Sun}$ (but most of it is absorbed internally!); in the next slides we try to understand why

Degenerate gas

- When density is large enough, inter-particle interactions and quantum effects cannot be neglected
  - $\Rightarrow$ Perfect gas approximation is not valid
- Here we consider the case of electrons
  - Highly ionized gas $\Rightarrow$ we consider free electrons
- A degenerate gas (or Fermi gas) is composed of *fermions* in identical (= degenerate) quantum states
  - Quantum state of a free electron is determined by spin (↑/↓) and 6 variables: x, y, z, p_x, p_y, p_z; but remember $\Delta x \Delta p_x \sim \hbar$
  - Pauli exclusion principle: maximum one particle per each *distinguishable* state $\Rightarrow$ 2 electrons per $\hbar^3$ hyper-volume

When this quantum effect becomes significant, many electrons are forced to have a larger momentum than expected for an ideal gas



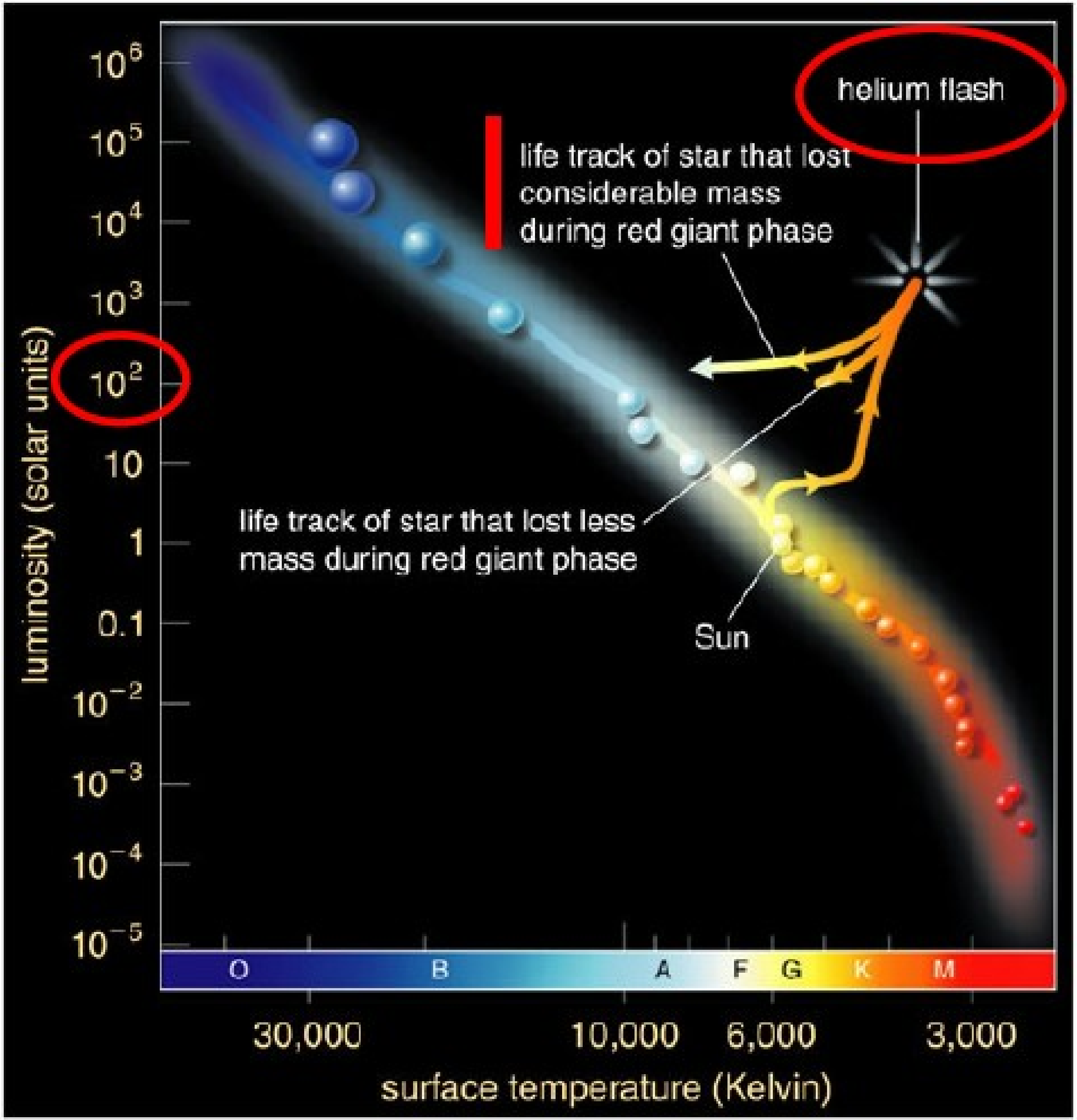
Why does it give a flash?

- Runaway phenomenon:
  - Electrons have a larger average momentum (⇒pressure) than for an ideal gas
  - Pressure now depends very little on temperature
  - As Helium fuses, T increases (⇒more fusion) but P is <u>not</u> increasing and it cannot compensate
  - ⇒ No hydrostatic equilibrium anymore
- It ends when:
  - Temperature becomes so high that quantum effects are again negligible
  - Core expands ⇒ core less dense, and cooler

Why not for $M/M_{Sun} > 2.25$?

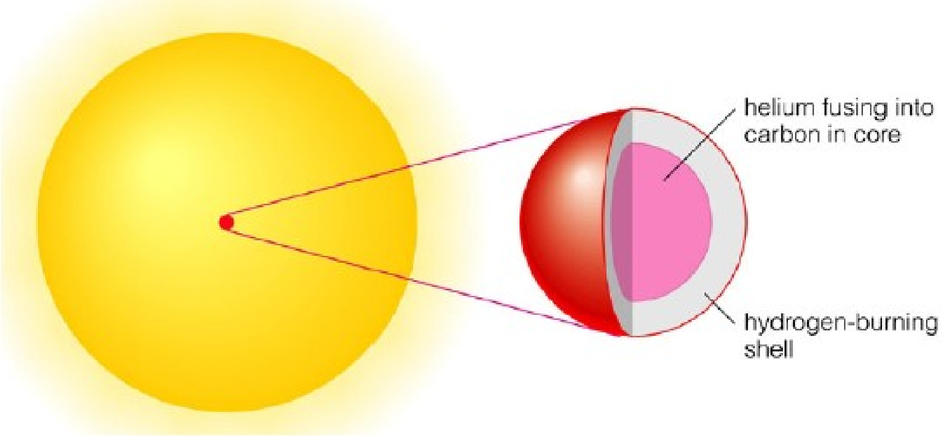
- A star with mass greater than about 2.25 solar masses starts to burn Helium without its core becoming degenerate
- Hot enough that He core is never allowed to contract to the point of degeneracy
- Exact value of this mass threshold depends on "metallicity" (i.e., in astrophysics jargon, the fraction of atoms that are not H or He, including non-metallic elements)
- *(We will not calculate it here, but it can be a possible topic for your dissertation)*

What happens next (1)

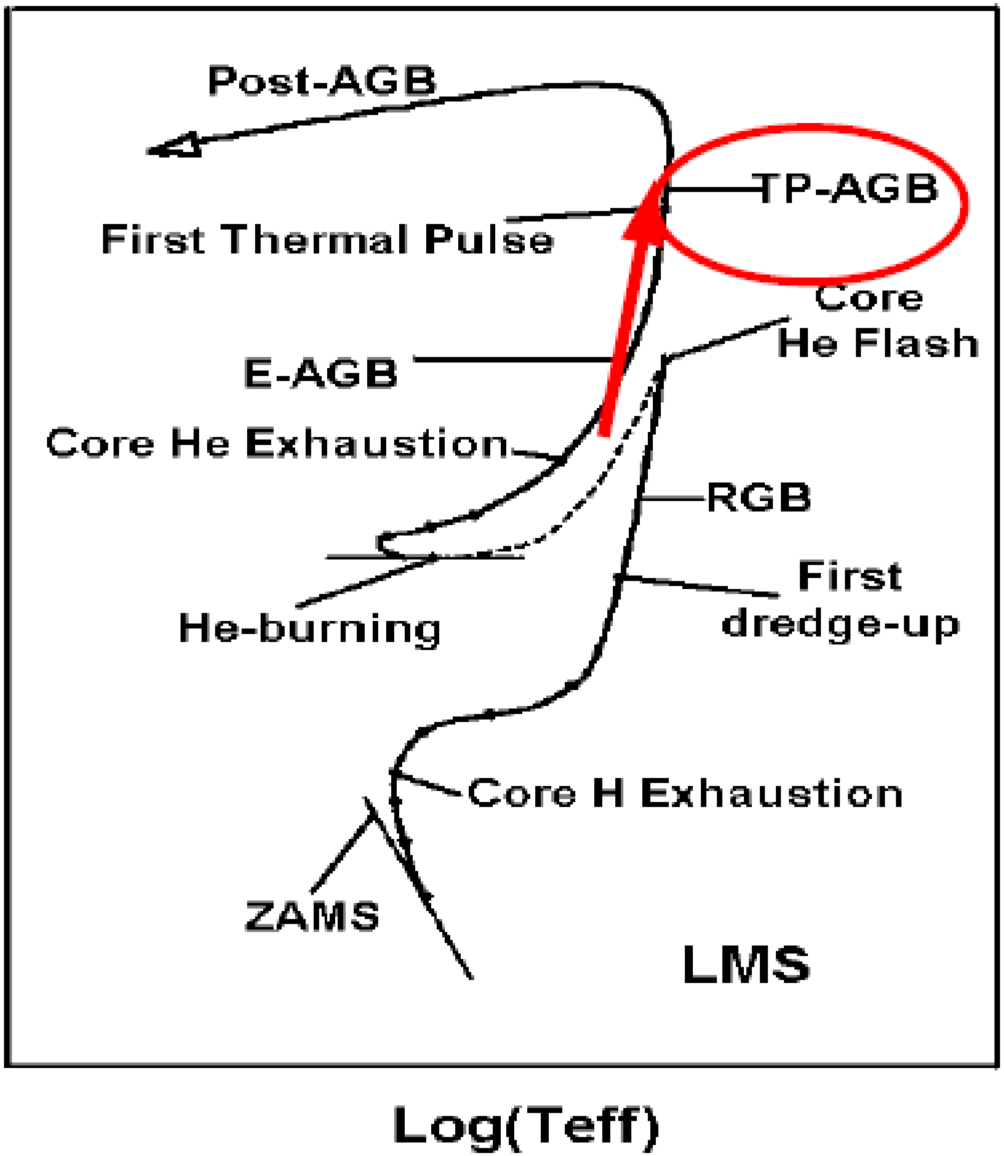


Horizontal branch star
(~100 million years)

- Stable combustion of He
- Core is not degenerate
- Temperature increases



What happens next (2)

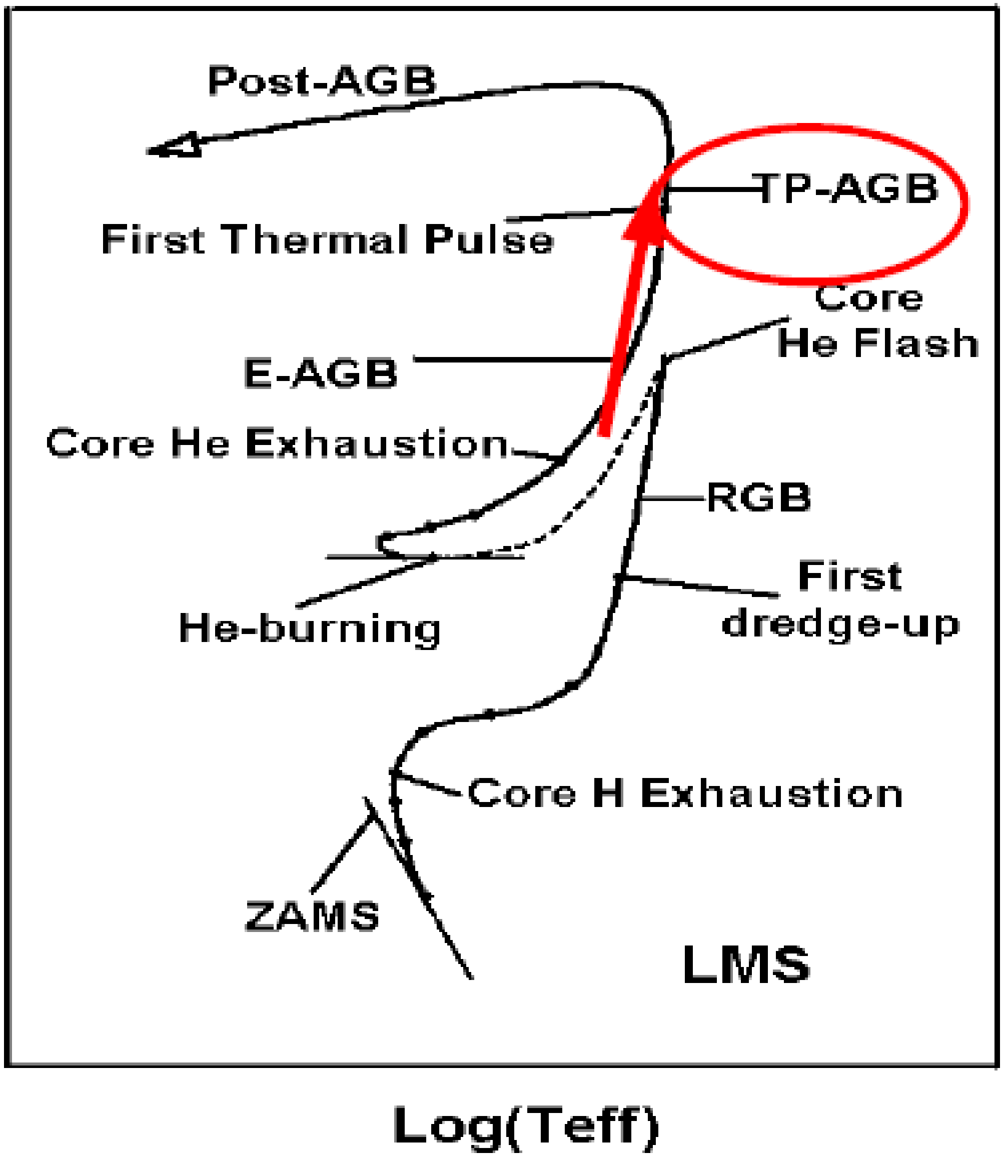


Thermal pulses:

- $\varepsilon(3\alpha) \sim T^{40}$
- $\Rightarrow$ Small perturbations in T cause huge differences in reaction rate and energy yield
- $\Rightarrow$ Instability $\Rightarrow$ pulsation

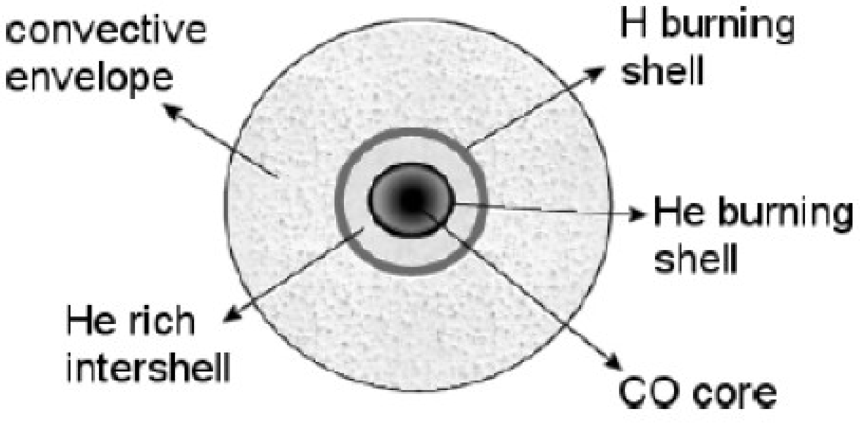
AGB: Asymptotic Giant Branch; it starts when no more He in core

What happens next (3)



Thermal pulses:

- O(1000) years: H burning
- O(1) years: He-shell flash
- → Strong expansion
- → stop of H burning
- O(100) years: He burning
- → strong increase in T
- → H burns again



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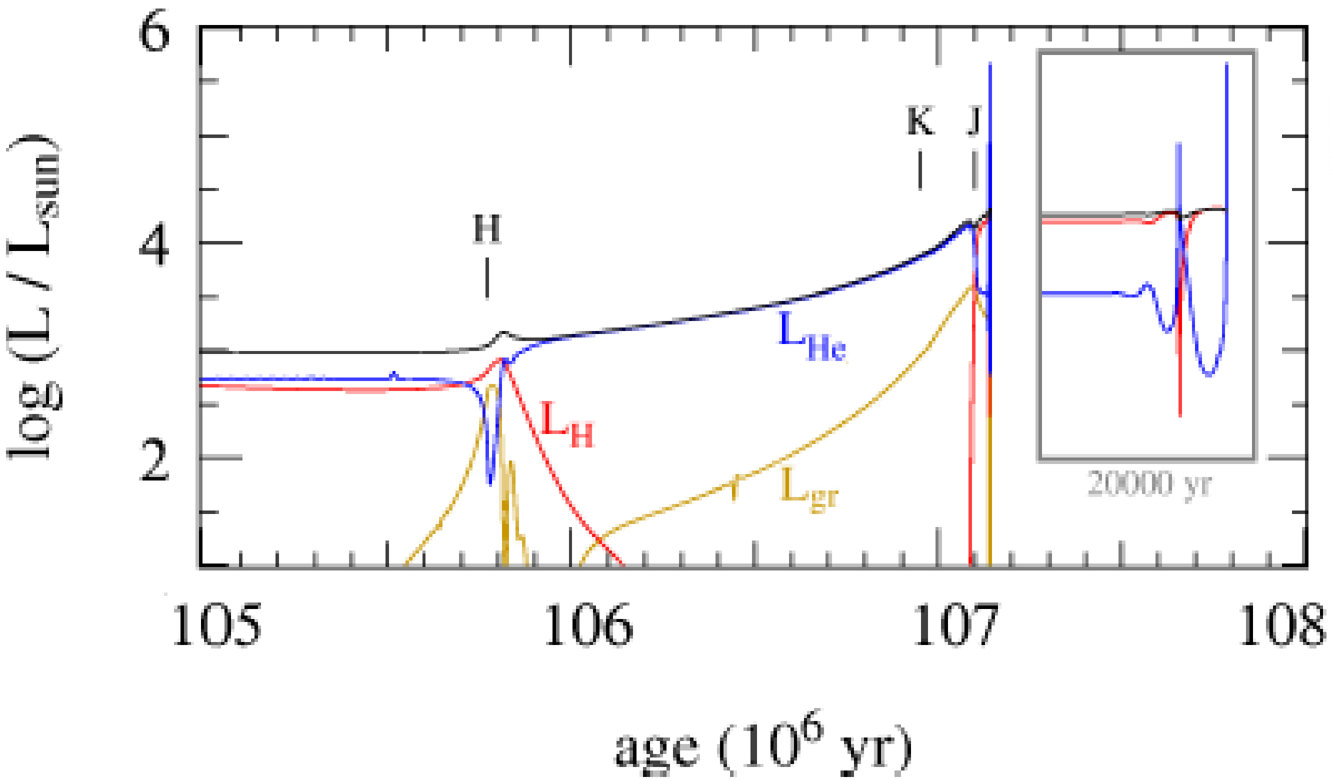
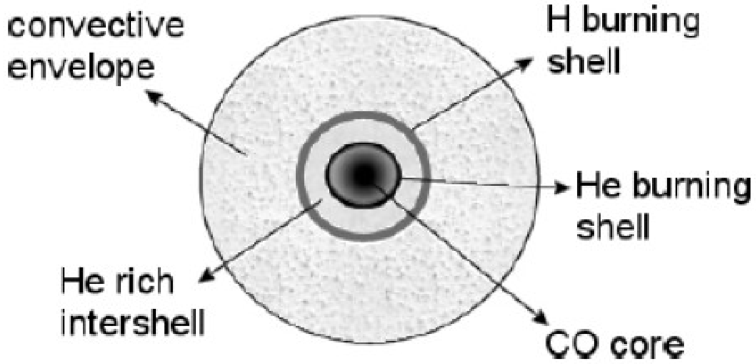
From early AGB to thermal pulses




convective envelope

H burning shell

He burning shell

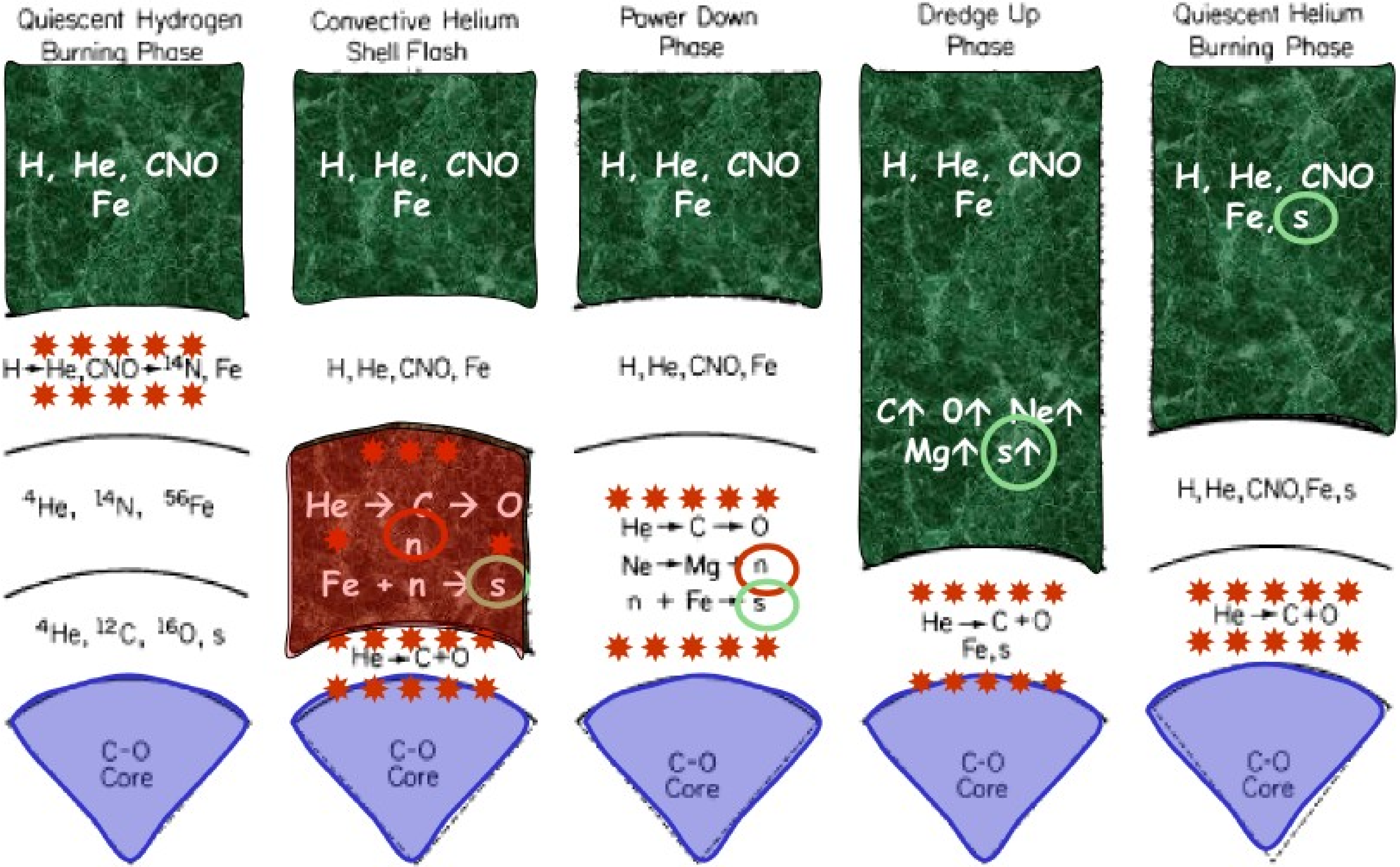
He rich intershell

CO core

Early AGB phase starts at point H: He burning shifts suddenly from the core to a shell; H-burning shell extinguishes.
H-burning shell is re-ignited at point J (start of thermal pulses).

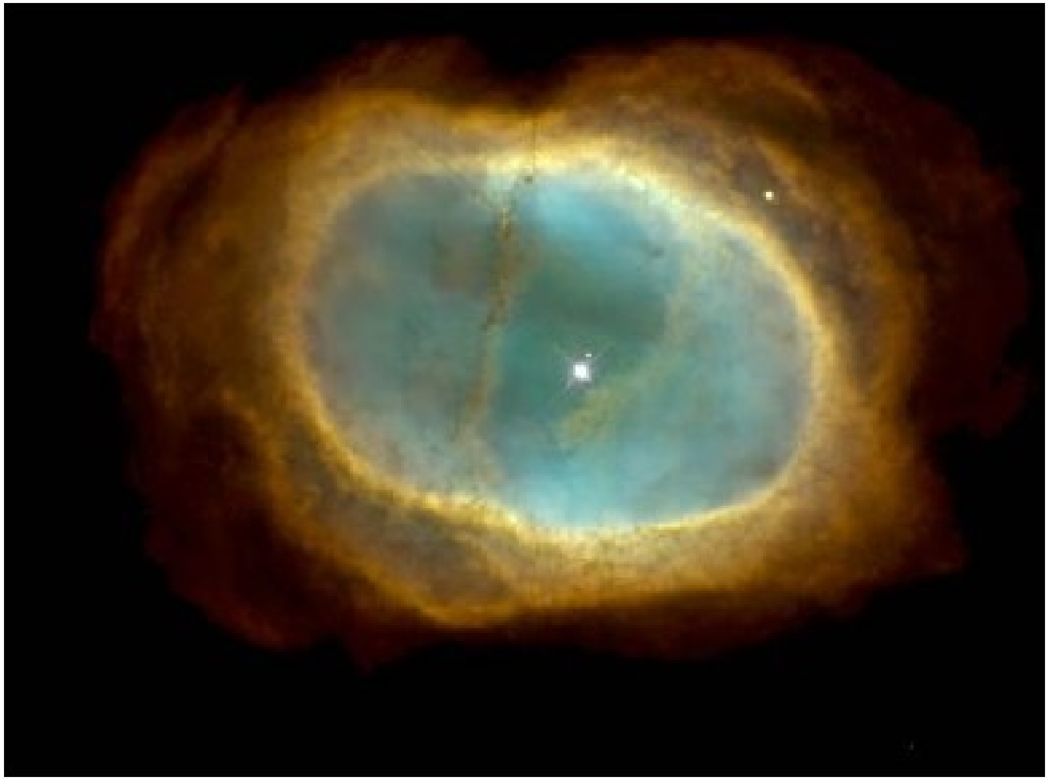
Nucleosynthesis in AGB stars

- Most striking proof of active nucleosynthesis in AGB: presence of Technetium, which has no stable isotope
  - Longest-lived, ^{99}Tc, has a lifetime of $2x10^5$ years
- Main source of neutrons: $^{13}C(\alpha,n)^{16}O$ reaction
- Then "slow" neutron capture creates heavier elements (slow with respect to β-decay)
  - This is called "s-process"
  - Elements heavier than Fe are created, up to Pb
    - Note: Fe is the most stable element; fusion up to Fe *releases* energy, after Fe *absorbs* energy



So now we know why elements up to Pb exist today.
What about elements like U? A bit of patience...

Planetary nebula



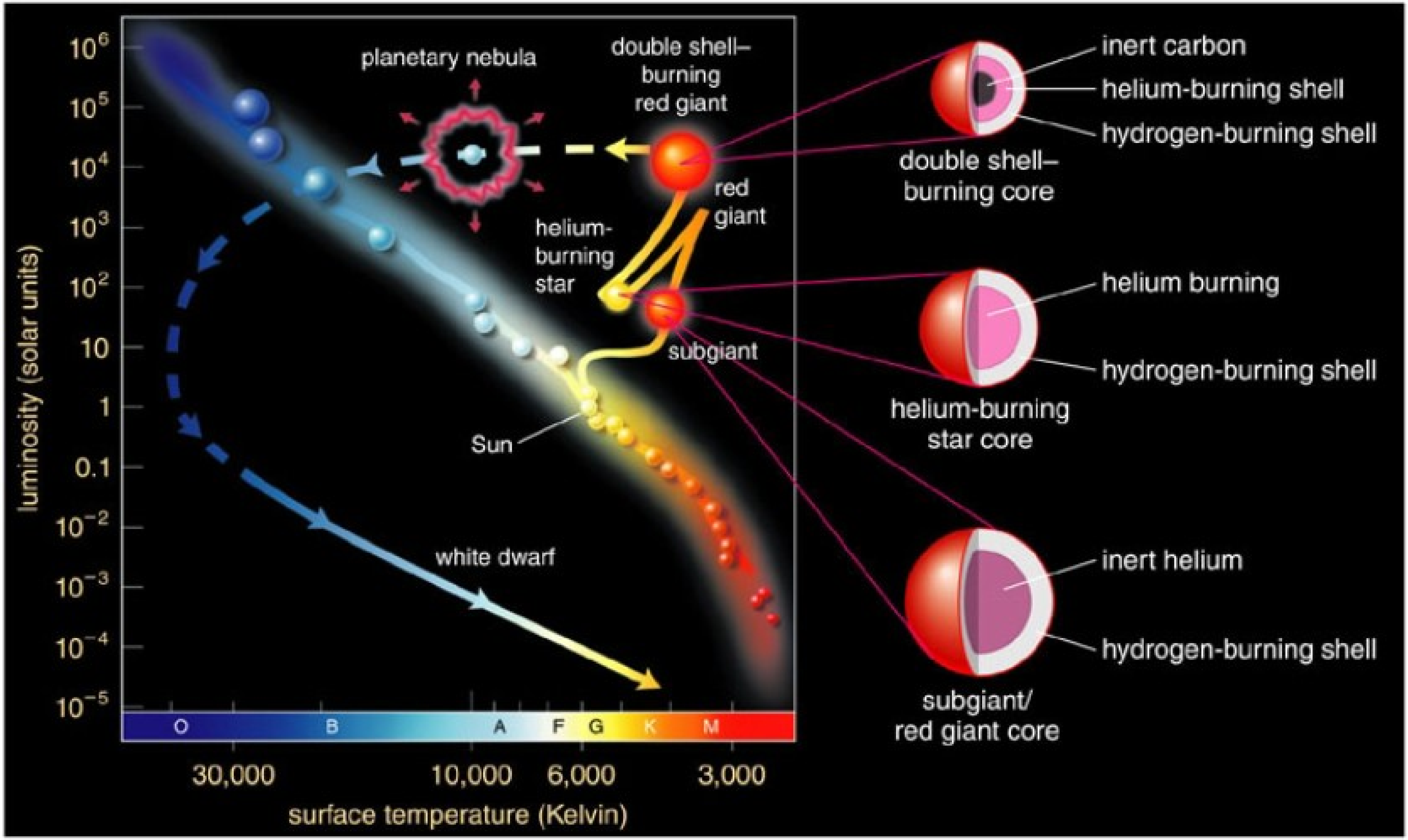
Pulsation causes a strong ejection of mass, which is further dispersed by radiation pressure, until all envelope is gone

The remnant is a White Dwarf at the center

White dwarfs

- All stars with masses up to $8M_{Sun}$ develop degenerate cores and lose their envelopes during the AGB phase
- They all end up as white dwarfs
- Nuclear fusion no longer provides energy
- They shine due to gravitational contraction, cooling at almost constant radius and loosing luminosity
- Collapse halts (if $M_{dwarf}<1.4M_{Sun}$) because of electron degeneration, when $R{\sim}0.01R_{Sun}{\sim}R_{Earth}$
- They cool off completely in $\sim 10^{13}$ years (notice that Universe is $\sim 10^{10}$ years old)

- *White dwarf properties can be subject of a dissertation*



planetary nebula
double shell–burning red giant
red giant
helium-burning star
subgiant
Sun
white dwarf
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
surface temperature (Kelvin)
inert carbon
helium-burning shell
hydrogen-burning shell
double shell–burning core
helium burning
hydrogen-burning shell
helium-burning star core
inert helium
hydrogen-burning shell
subgiant/red giant core

① **Protostar:** A star system forms when a cloud of interstellar gas collapses under gravity.

② **Yellow main-sequence star:** In the core of a low-mass star, four hydrogen nuclei fuse into a single helium nucleus by the series of reactions known as the proton-proton chain.

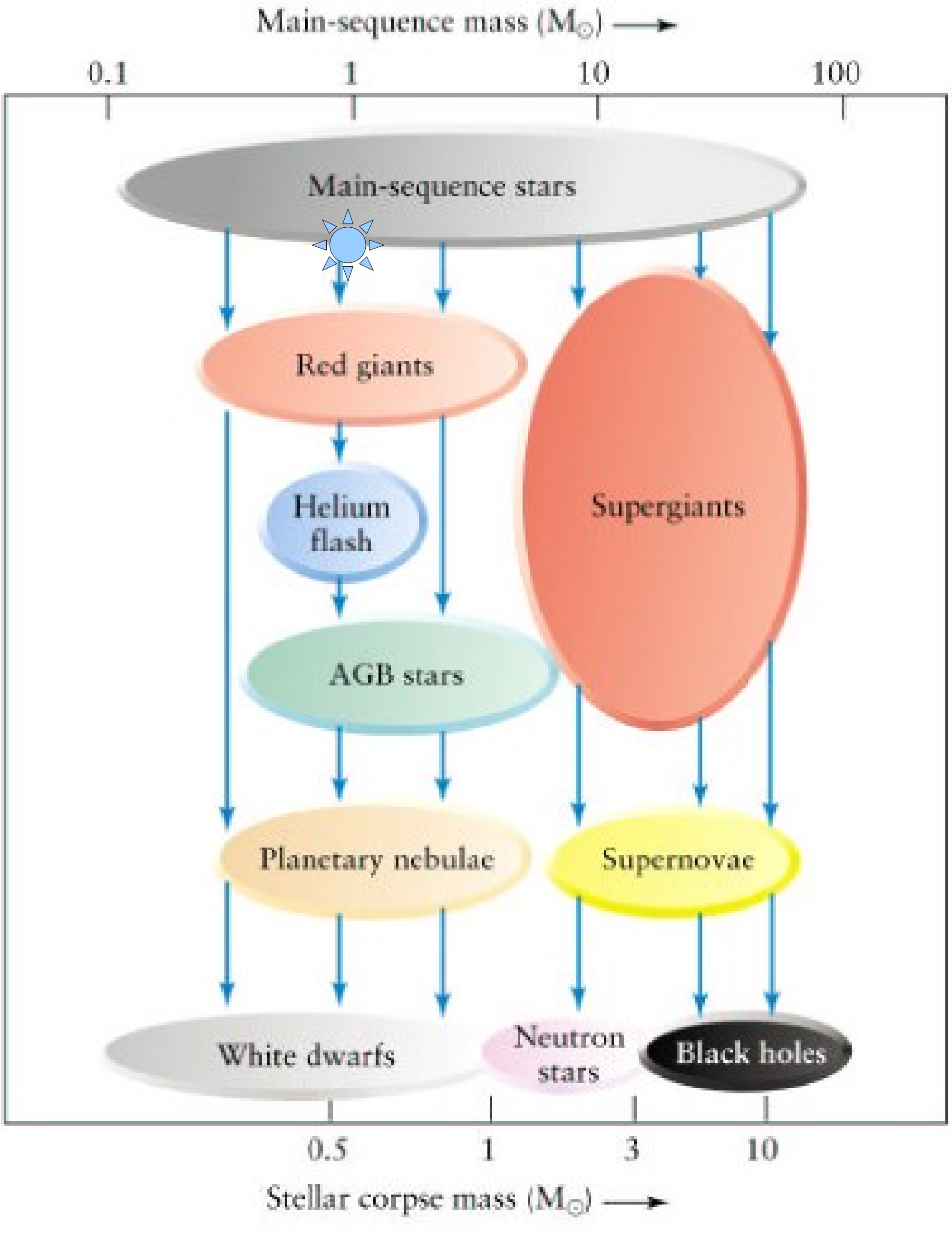
③ **Red giant star:** After core hydrogen is exhausted, the core shrinks and heats. Hydrogen shell burning begins around the inert helium core, causing the star to expand into a red giant.

④ **Helium-burning star:** Helium fusion begins when the core becomes hot enough to fuse helium into carbon. The core then expands, slowing the rate of hydrogen shell burning and allowing the star's outer layers to shrink.

⑤ **Double shell-burning red giant:** Helium shell burning begins around the inert carbon core after the core helium is exhausted. The star then enters its second red giant phase, with fusion in both a hydrogen shell and a helium shell.

⑥ **Planetary nebula:** The dying star expels its outer layers in a planetary nebula, leaving behind the exposed inert core.

⑦ **White dwarf:** The remaining white dwarf is made primarily of carbon and oxygen because the core of the low-mass star never grows hot enough to produce heavier elements.


Main-sequence mass ($M_\odot$) ⟶
0.1
1
10
100
Main-sequence stars
Red giants
Helium flash
AGB stars
Supergiants
Planetary nebulae
Supernovae
White dwarfs
Neutron stars
Black holes
0.5
1
3
10
Stellar corpse mass ($M_\odot$) ⟶