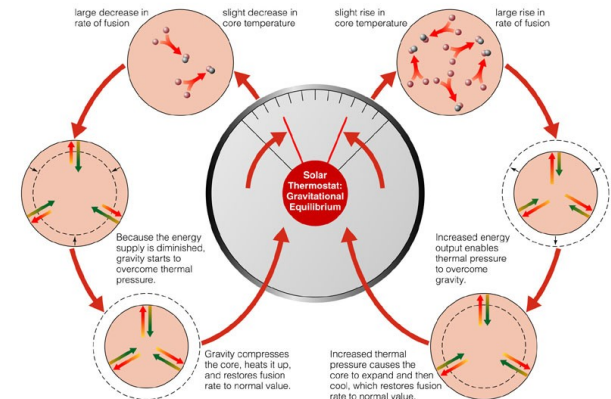
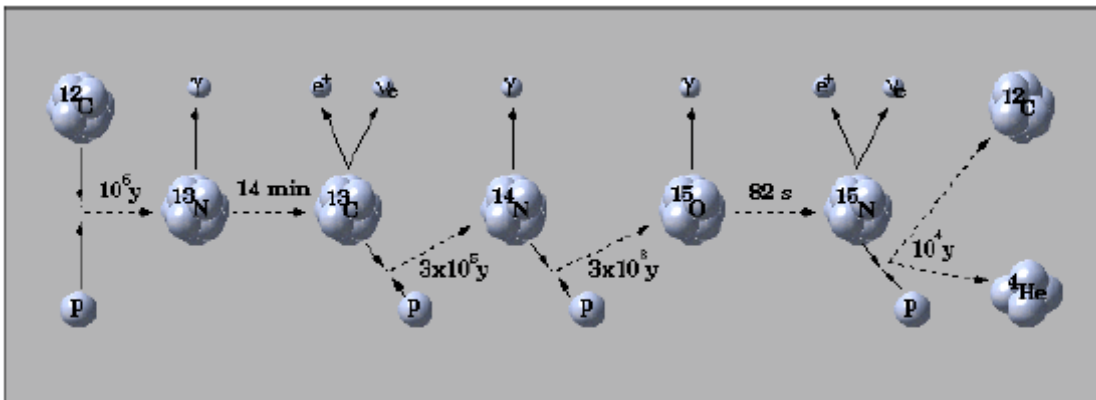
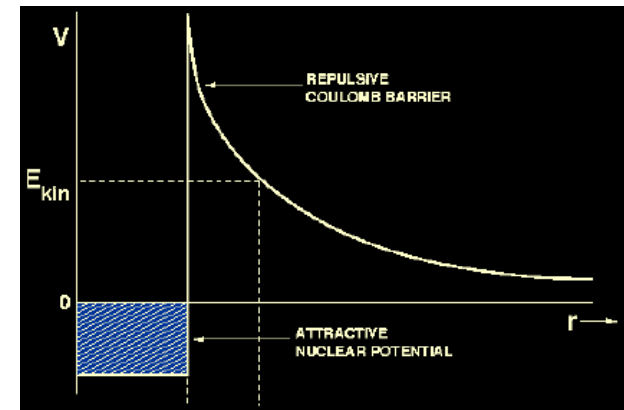


Astrophysics and Nuclear Astrophysics (LPHY2263)

Andrea Giammanco, UCL

Chapter #3

- Introduction to nuclear astrophysics
 - Sun's lifetime
 - How can nuclear fusion happen
 - Main nuclear processes
 - Solar neutrinos



Why do stars shine?

- Stars emit light because they are hot, for the same principle as a traditional lamp bulb
- As we have seen in previous lessons, the light spectrum from a star is very close to an ideal "black body" spectrum
- Luminosity = rate of energy loss
- In the next slides we will discuss what compensates for this energy loss and keeps the stars shining



Exercise #1

- Helmholtz calculated the **Sun's lifetime** under the hypothesis that it is made of burning coal
 - Not because he thought it could be made of coal, but because this could be easily generalized to any chemical process, to show that Sun's energy cannot be of chemical origin
- Repeat his calculation, using his inputs:
 - Coal: 3.0×10^7 J/kg of potential energy of combustion
 - Mass of the Sun: 2.0×10^{30} kg
 - Energy output of the Sun: it was thought, at that time, to be 3.6×10^{26} J/s

Homework #1

- If there is no internal source of energy, **how long can a star shine?** (First calculated by Kelvin and Helmholtz)
 - No internal source of energy implies that there is negligible internal pressure
 - (By the way: would pressure be exactly 0?)
 - **Energy output comes from contraction under gravity**
- Assume that L , M , R are known; derive lifetime
 - Do the exercise for the Sun
 - *Note: this can be chosen as a dissertation for the final exam. I can provide some help and additional material.*
 - ([Link](#) to some useful discussion)
 - Answer: ~100 million years
 - (but then geologists found that Earth is older than that)

How long can the Sun live?

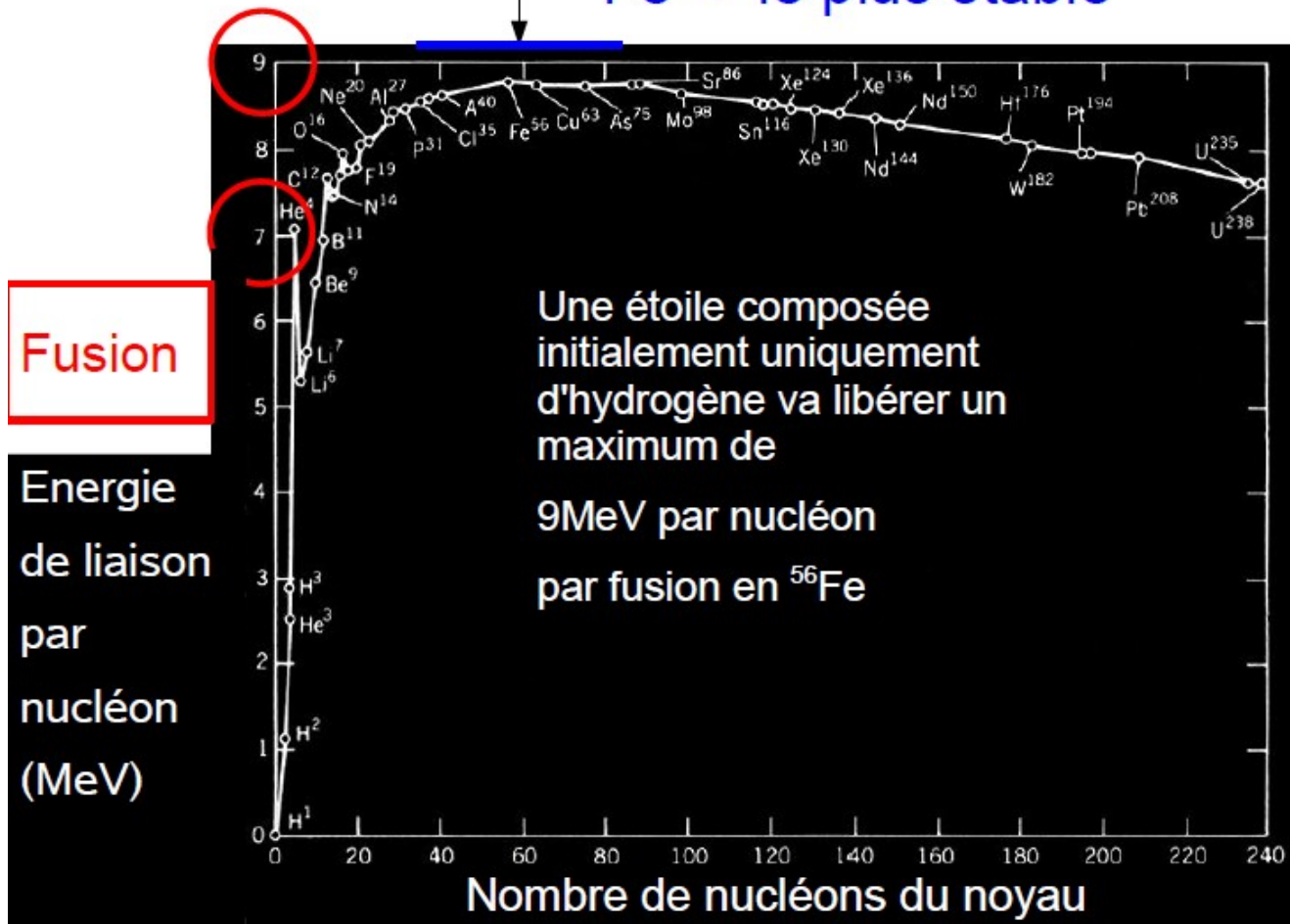
- Before talking about what can transmute 4 nuclei of ^1H (4p) into 1 nucleus of ^4He (2p2n), let's consider what happens if 1 gram of ^1H entirely becomes ^4He
- Because of the binding energy (see next slide), it means that you get 0.993 grams of ^4He
- The rest becomes $E=mc^2=6.3\times 10^{18}$ ergs
- Luminosity of the Sun is 4×10^{33} ergs/s
 - \Rightarrow ~4 million tons of matter convert into energy every second
 - \Rightarrow ~600 million tons of H fuse into He every second
 - Sun contains $\sim 10^{21}$ million tons of H
 - Only part of it (~10%) is hot enough for fusion (as we will see later)
 - \Rightarrow **~10 billion years** before all this fuel is exhausted

Binding energy

- Nucleus with Z protons and N neutrons:

$$Q(Z,N) = (Zm_p + Nm_n - m_{\text{nucleus}})c^2$$
- Normalize by number of "nucleons" (p or n): $Q(Z,N)/(Z+N)$

Fe – le plus stable



Fission

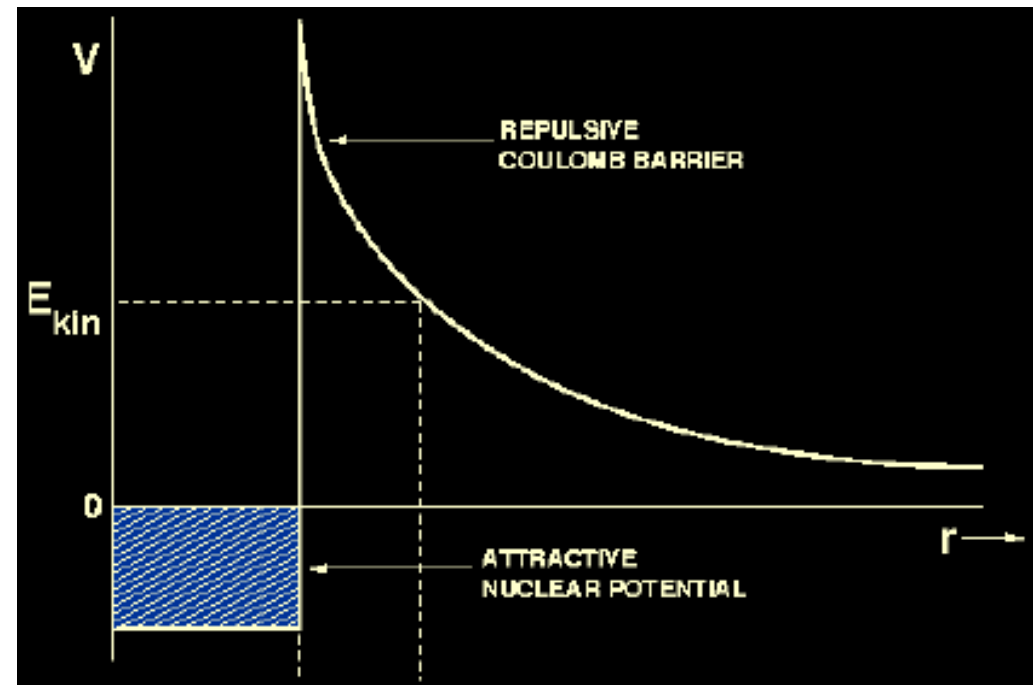
Note how fusion is, in principle, much more effective than fission in producing energy. Fusion bombs (H bomb) are more devastating than fission bombs (A bomb).

Nuclear fusion in the stars

- Now let's talk about what can transmute four protons into one α particle (i.e., He^4 nucleus)
- There is no *direct* mechanism: it is highly unlikely that four protons collide simultaneously
- We also need two protons to become neutrons
 - The only fundamental interaction that can do that is the "weak nuclear force" (*please come to the blackboard*)
 - Normally it works the other way around: a free neutron decays spontaneously (with a lifetime of 11') as $n \rightarrow p e^- \bar{\nu}$ (+ energy corresponding to the small mass difference)
 - (Question: why did I specify "a *free* neutron"?)

Coulomb barrier

- Just by knowing that nuclei exist, we know that an attractive force binds together the "nucleons" (protons and neutrons)
 - It is called "strong nuclear force"
 - At the nucleon level it is a short-range interaction (not for quarks)
 - *(Analogy: quark-nucleon like atom-molecule; more at the blackboard)*
- But there is also a repulsive force between protons: electromagnetism, which has a long range ($F \propto e^2/r^2$)
- At small distances the strong force wins, but before you get there, EM repulsion dominates
 - Large kinetic energy needed to overcome this barrier, i.e., large temperature of the system

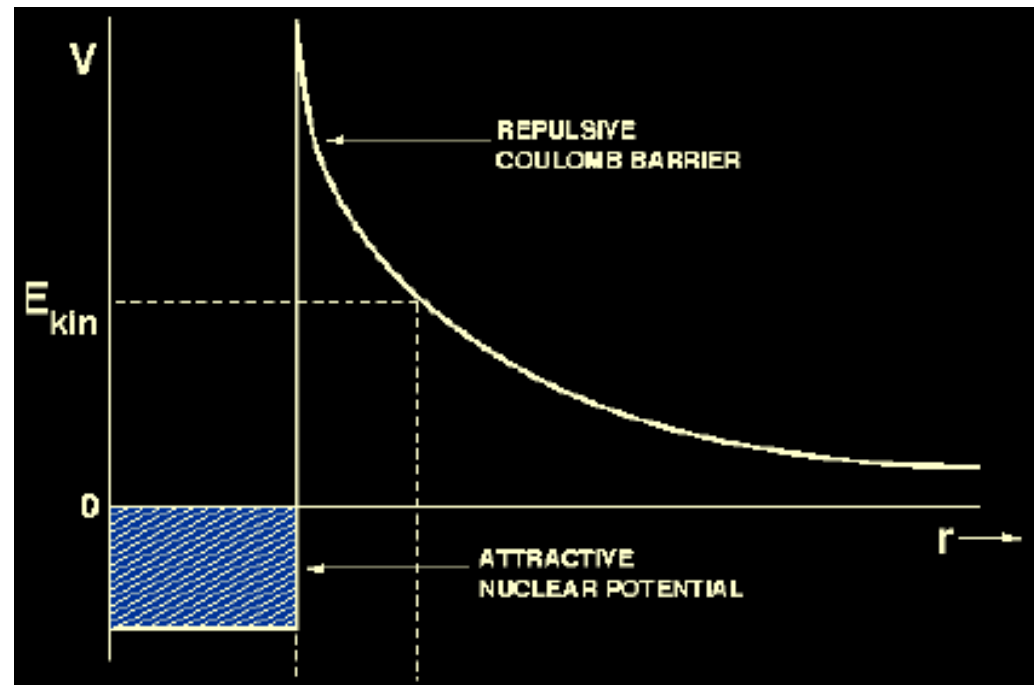


Note: the exact form of the nuclear potential is quite more complex, but this approximation can be used for our purposes here

Coulomb barrier

- $\langle E_{\text{kin}} \rangle = 3kT/2$ (k: Boltzmann const.)
- The Coulomb barrier is of the order of MeV
- \Rightarrow One would need $T \sim 10^{10}$ K
- Temperature in the middle of the Sun is $T \sim 10^7$ K
- \Rightarrow The average energy of the stellar gas is of order keV
- In the tail of the spectrum:
 $N_{\text{nucleons}} \propto e^{-E/kT}$
- A barrier 1000 times larger than the average is passed by a fraction e^{-1000} ($=10^{-434}$)

- Number of nucleons in the Sun:
 $M_{\text{sun}} / M_n \sim 2 \times 10^{30} / 1.67 \times 10^{-27} \sim 10^{57}$
- How is it possible that nuclear fusion can happen at all?



Questions

- What is the strongest approximation I implicitly made so far?
- Is it a valid one?

Tunnel effect

- Intuitive explanation using Heisenberg uncertainty principle: $\delta E \delta t \sim h/4\pi$
- During the time δt of the interaction, the nucleon's wavefunction is not in a defined state of E , and a continuum distribution of energies is available
- Therefore, overcoming a potential barrier is allowed, even when classically impossible
- To avoid confusion: there is a distribution of "classical" E , from the thermal law, convoluted with the quantum effect

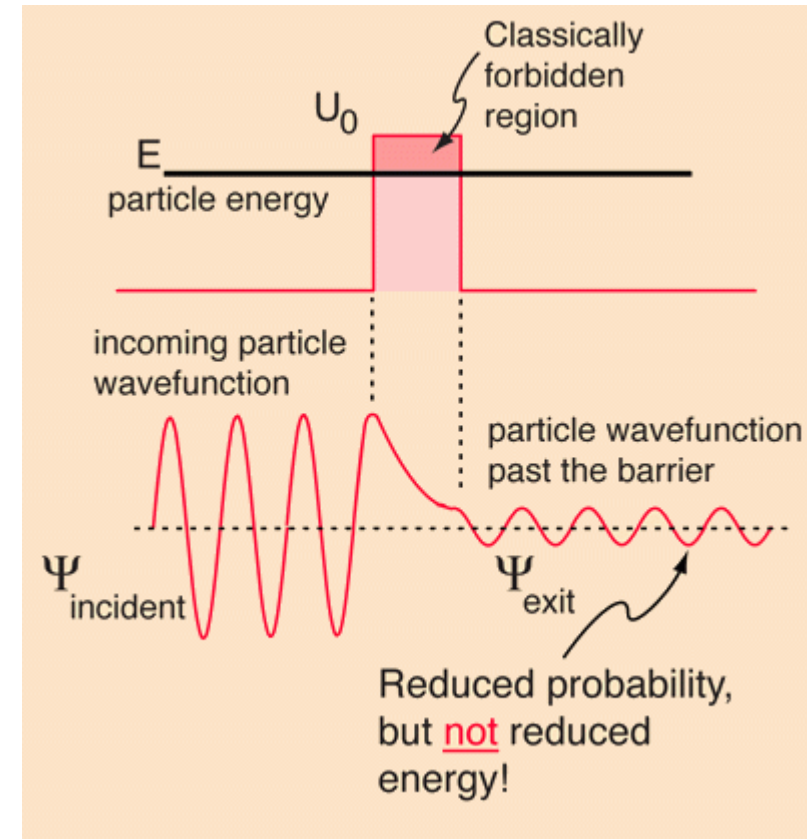
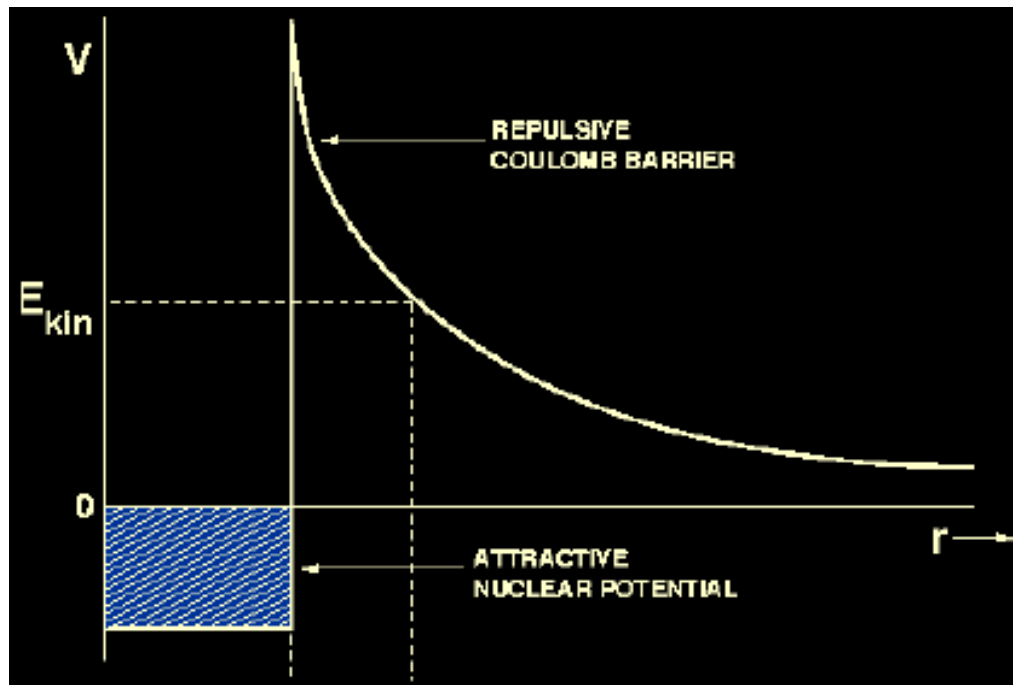


Image from [here](#)

Homework #2

- Calculate the **height** of the Coulomb barrier for protons, and the **transmission probability** at $E_{\text{kin}} \sim 10 \text{ keV}$

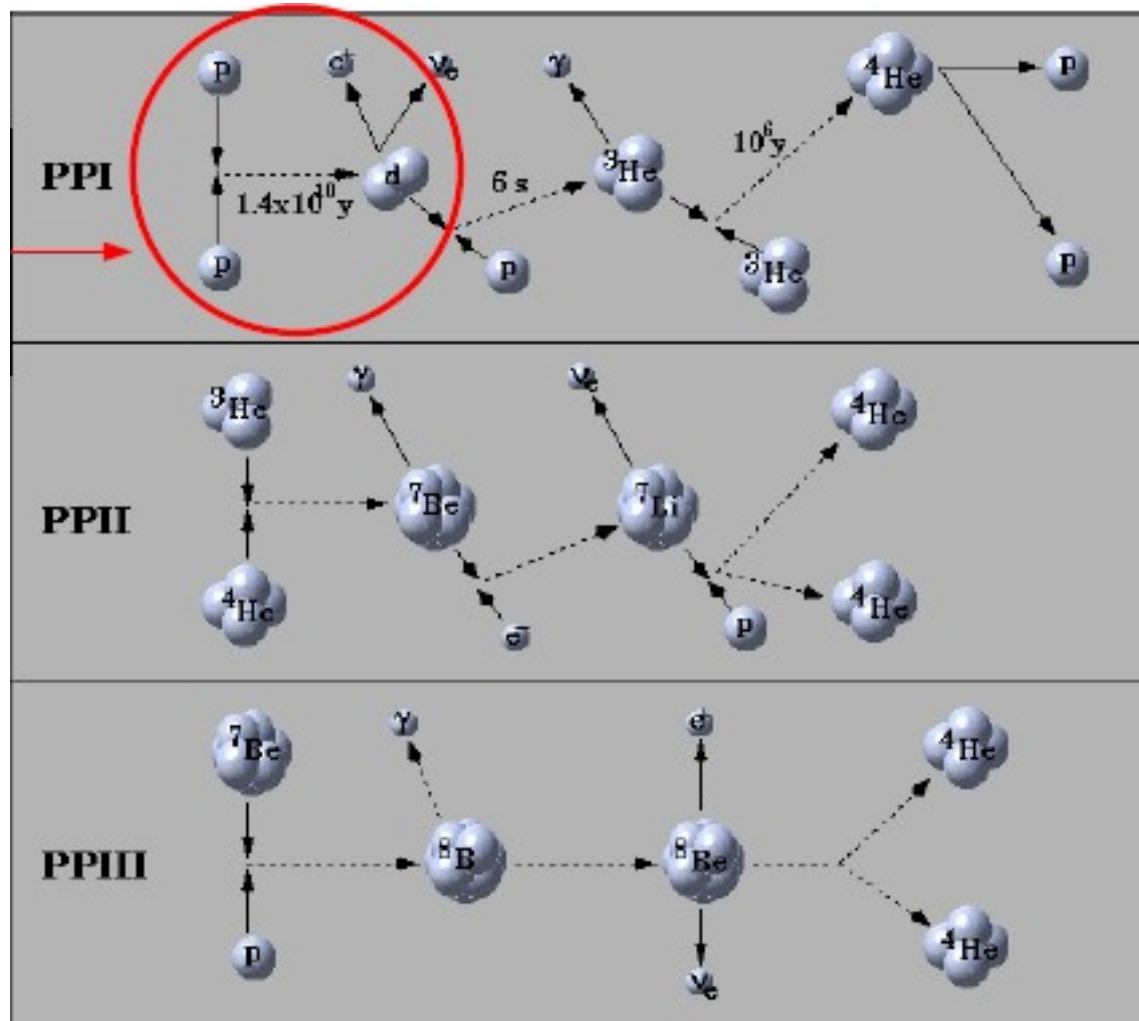
$$T = e^{-2 \int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2} (V(x) - E)}}$$



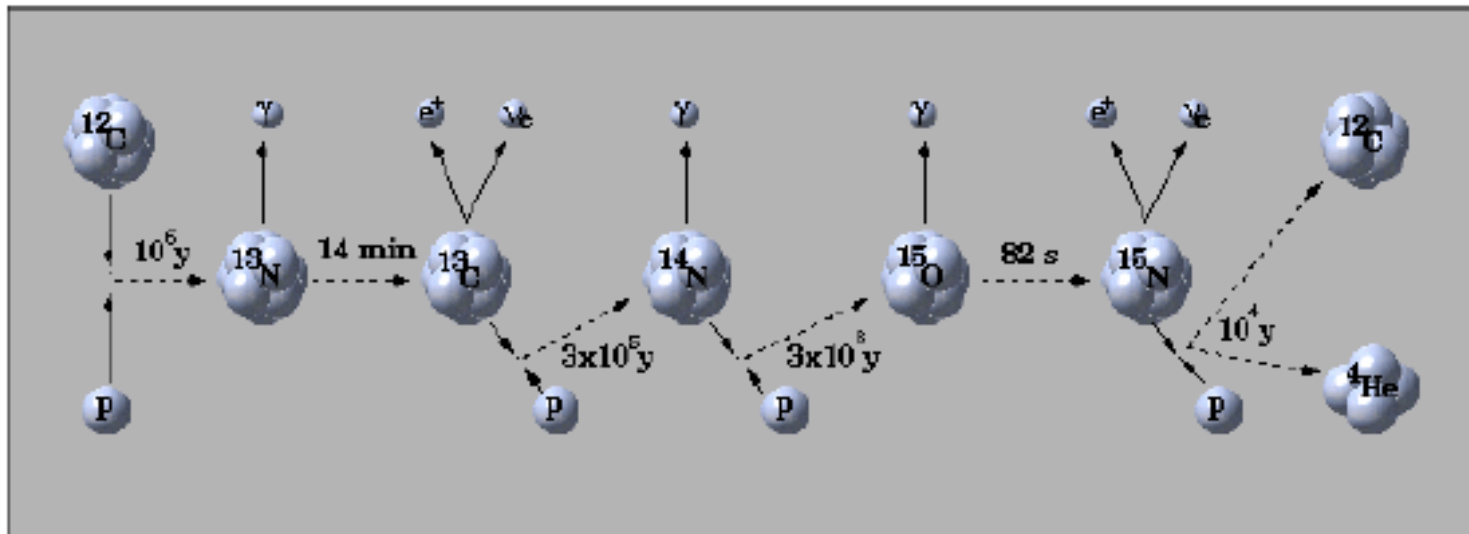
$x_1 \sim 10^{-15} \text{ m}$
 x_2 : you can calculate it

x_1 x_2

Proton-proton chain



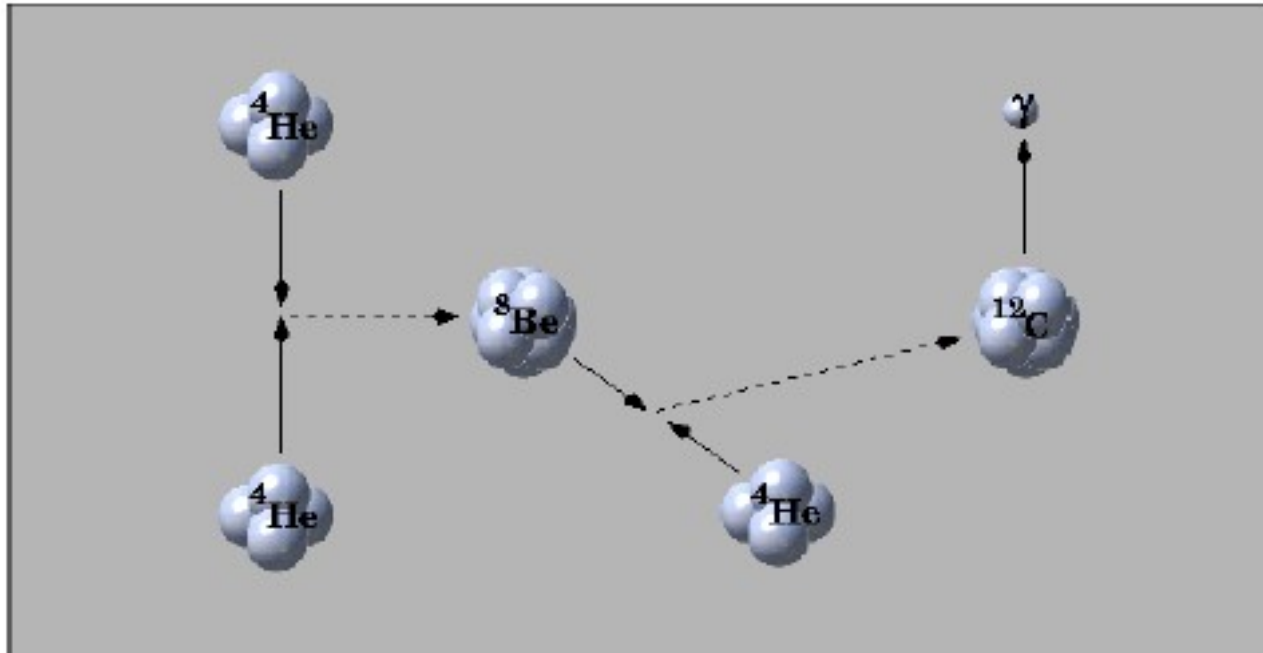
CNO cycle



Carbon acts as a catalyst: its number at the end is the same as at the beginning.

But where is C coming from?

Triple α reaction



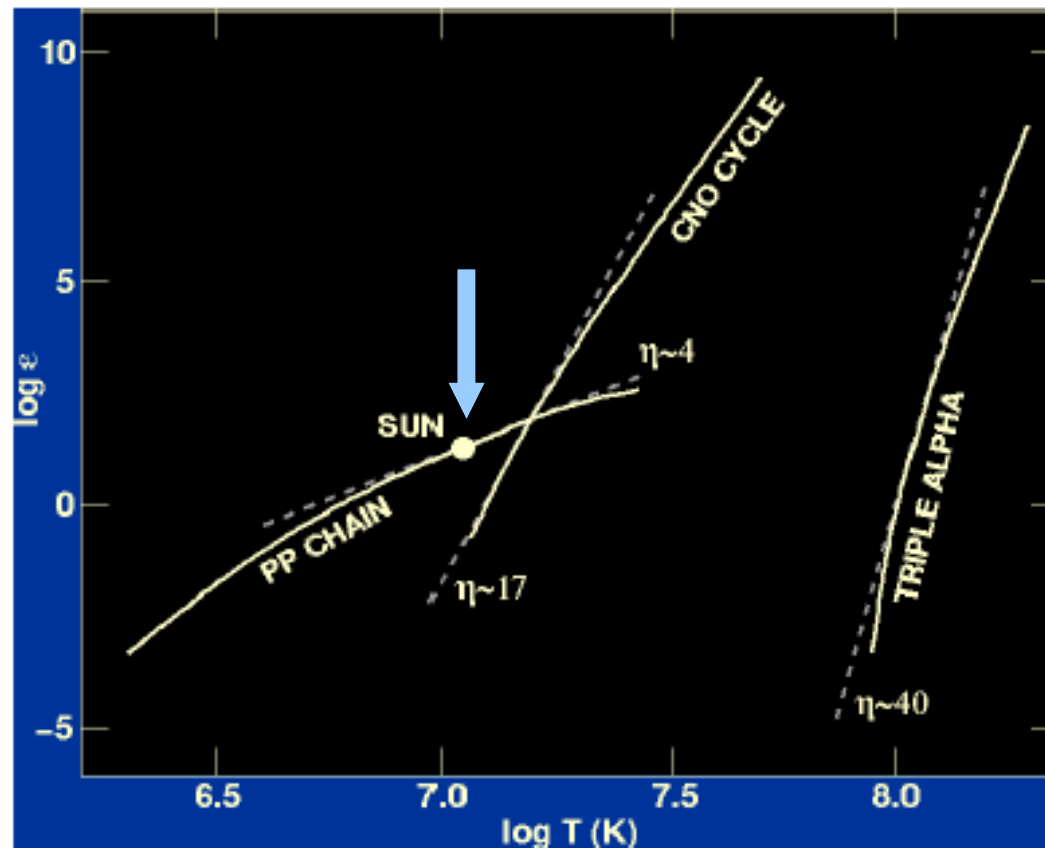
This reaction explains why C is much more abundant ($\sim 10^5$ times) than Li, Be, B, although it is heavier than them

Where is this happening?

- Different processes require different energies
- Gradient of temperatures from core to surface
- Different processes happen at different depths

Energy released
per unit mass
and unit time:

$$\epsilon \propto T^\eta$$

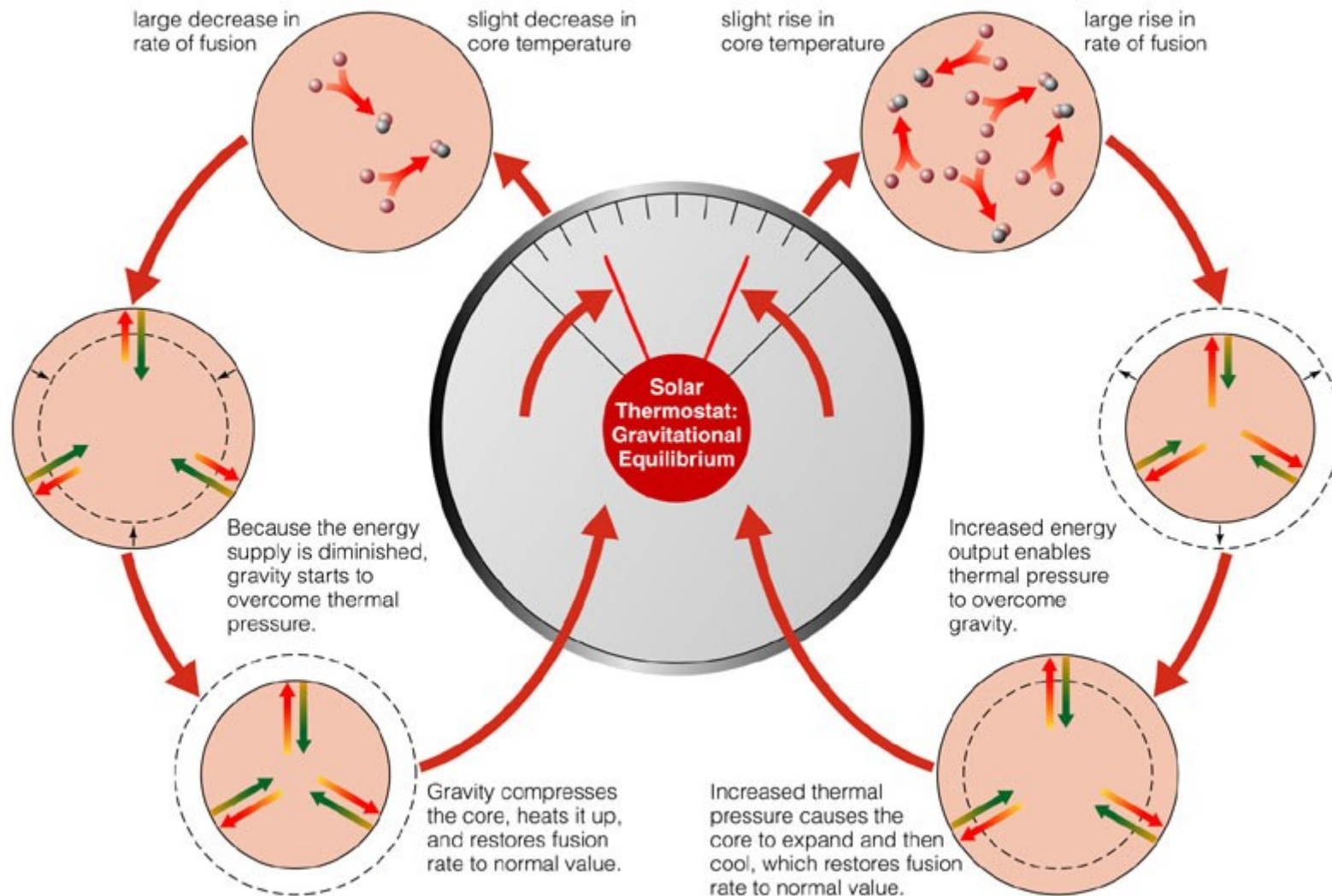


Note: the 3α reaction is negligible in the Sun \Rightarrow the ^{12}C that causes the CNO cycle is mostly coming from somewhere else (you will discover in a later lesson)

Dependence on temperature

- Fusion reaction rates depend on T
 - $\epsilon(\text{PP}) \sim T^4$
 - $\epsilon(\text{CNO}) \sim T^{17}$
- More fusion makes the core hotter
- Higher temperature in the core makes fusion more frequent
- Same principle as the H bomb
- So why doesn't the star explode like a bomb?

Hydrostatic thermostat

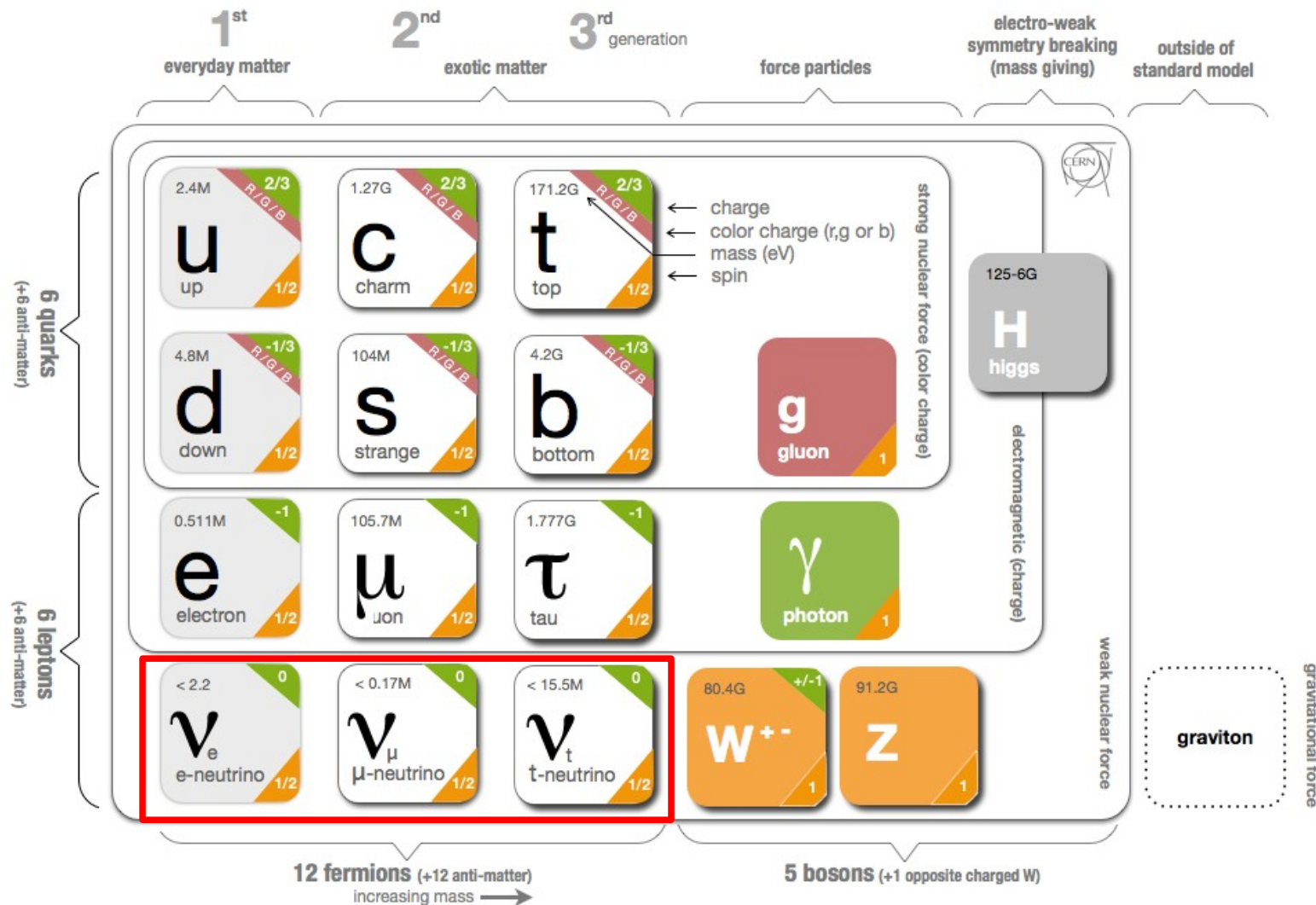


Recommended reading

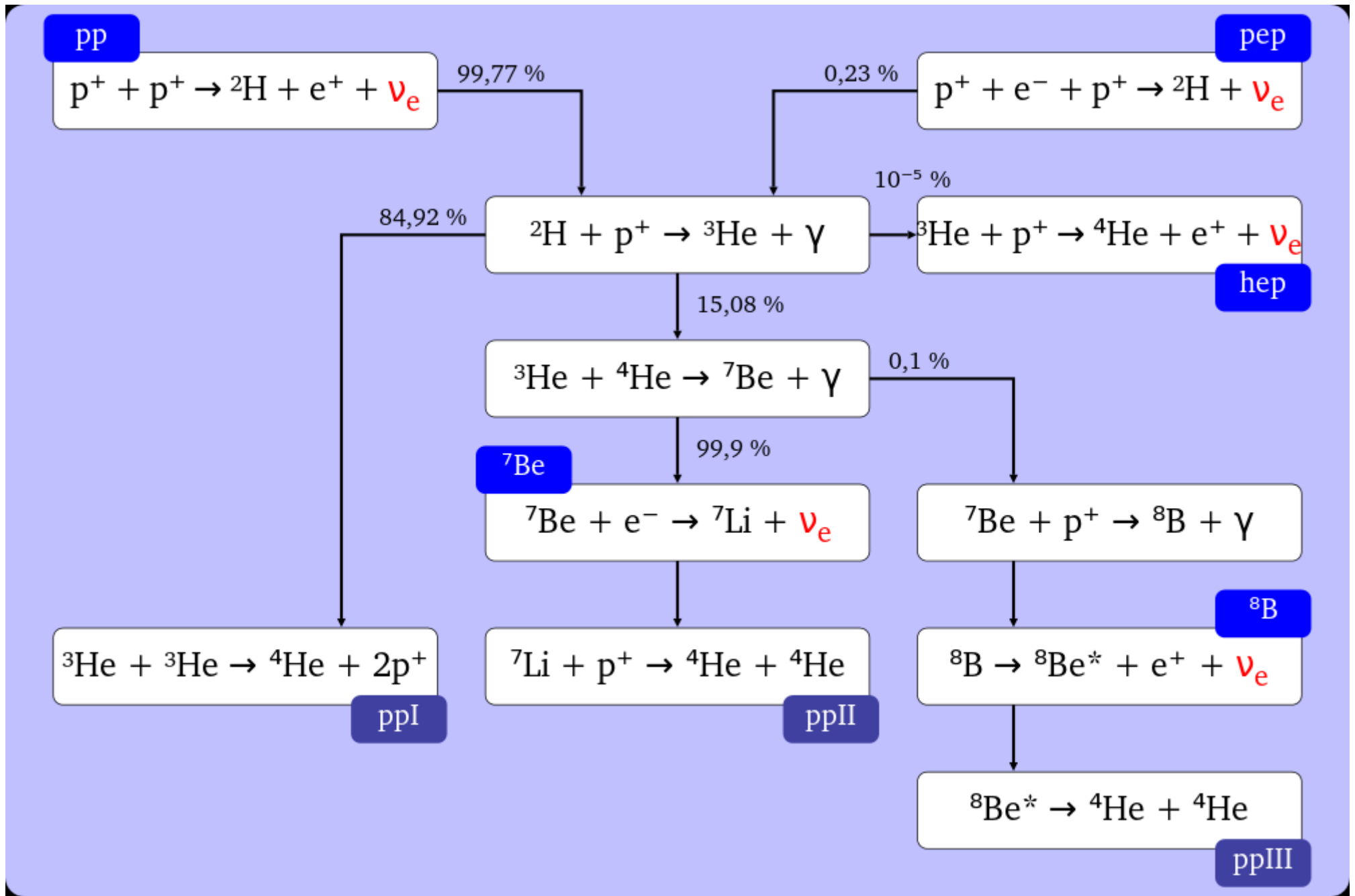
- H.Bethe, "Energy production in stars", Physical Review 55 (1938) 434
- Online:
<http://journals.aps.org/pr/pdf/10.1103/PhysRev.55.434>
- He suggested the main nuclear reactions responsible for stellar energy
- He obtained the mass-luminosity relationship in a more rigorous way (more accurate and more general)
- He demonstrated that elements heavier than He are too short-lived to have been produced in the Sun \Rightarrow the amounts that we have on Earth must come from somewhere else (Sun is a "second generation" star)

Neutrinos

Image from here



Neutrino properties: insensitive to EM and strong forces, sensitive to weak force, mass unknown but much smaller than anything else



From wikipedia

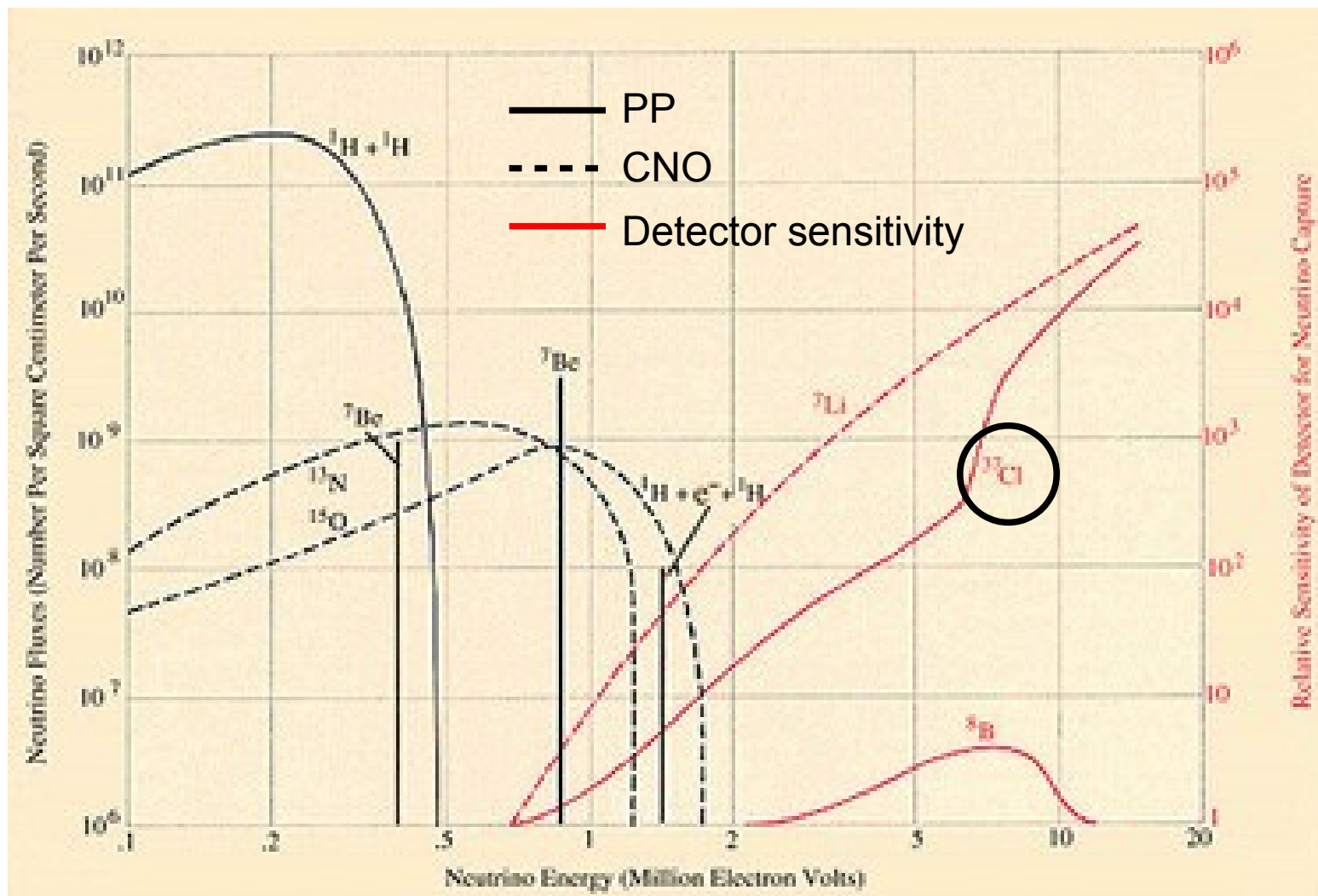
Solar neutrinos

- The stellar model that we have presented makes a **very precise prediction** of neutrino production from the known nuclear reactions
 - Flux at the Earth surface is 7×10^{10} neutrinos/cm²/s
- All of them, no exceptions, are "electronic neutrinos"
 - No anti-neutrinos (*Question: why?*)
 - No muonic/tauonic neutrinos
- Different processes give different kinetic energies to the neutrinos
 - Note: different detectors are sensitive to different energy thresholds, so they might be blind to the neutrinos from some of these reactions

Some neutrino properties

- Because they are almost massless, they travel almost at the speed of light
- Because they are neutral by both strong and EM interaction, and they feel only the weak force:
 - They can travel through the Sun at almost the speed of light, from the core (which is the part hot enough to induce fusion reactions) to the surface
 - (Note: instead, photons are very slow in the Sun! *Why?*)
 - It is very difficult to observe them: observing means making them interact with something!
 - To reduce neutrino flux to $1/e$ of its original value, you would need one parsec of steel

Solar neutrino spectra



Question: why are some spectra continuous and some discrete?
Dirac deltas, or just very narrow?

Solar neutrino observation

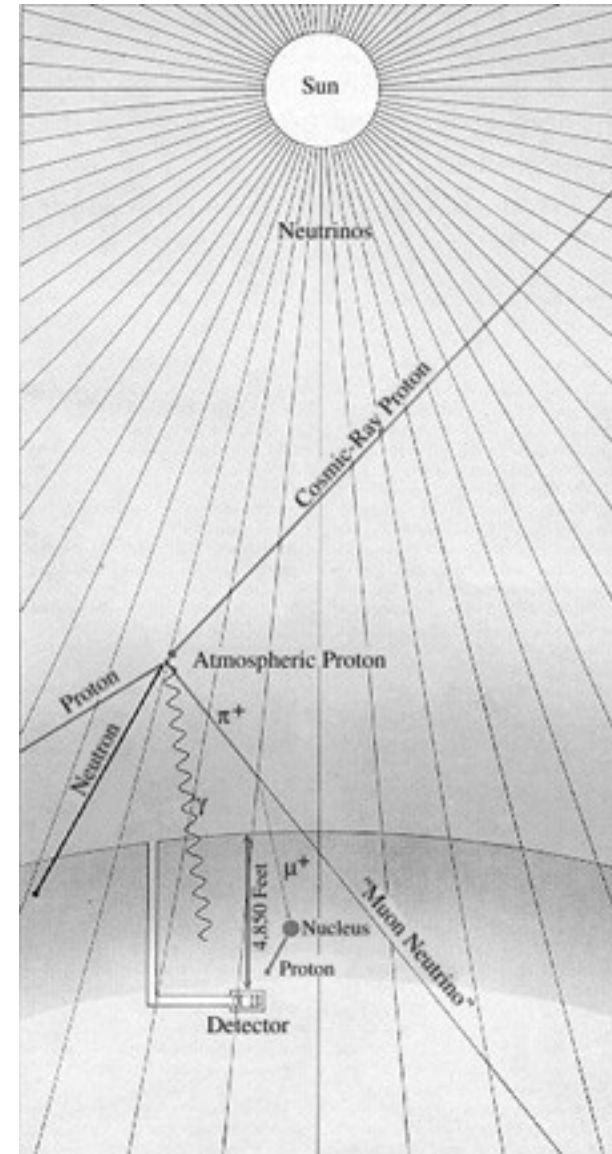
- Homestake Experiment, '60s ([link](#))
- Tank filled with C_2Cl_4 , in a deep mine
 - A cheap cleaning fluid
- Based on reaction $\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$
- By bubbling He gas through the liquid, ${}^{37}Ar$ atoms were extracted and counted regularly, using their β -decay
$${}^{37}Ar \rightarrow {}^{37}Cl + e^+ + \nu_e$$
- Threshold 0.814 MeV for the $\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$ reaction \Rightarrow experiment is sensitive only to ν from PP-II and PP-III



Image from Nobel prize site

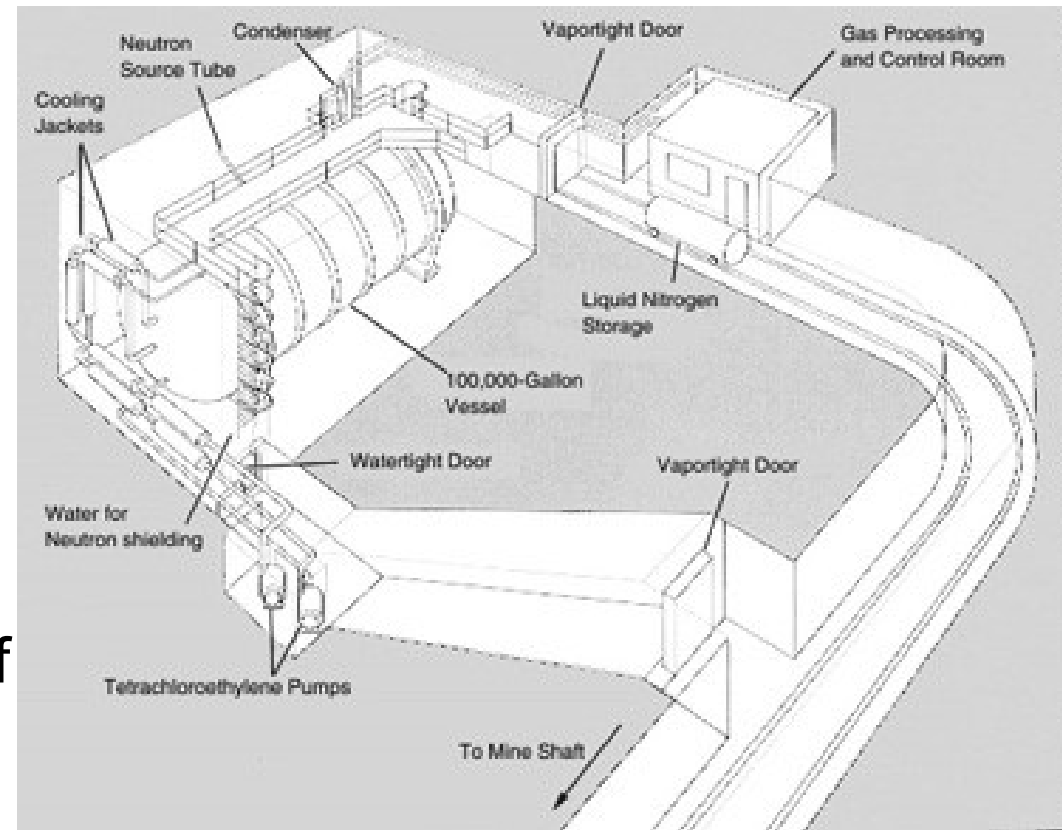
Cosmic background

- Cosmic rays will be the topic of a future lesson; here I only mention that they come from various sources, including the Sun itself; mostly protons; they have a very large flux and can reach very high energy
- What is relevant here is that they can cause interactions in the atmosphere and in the rock
- Because the detector is underground, most of the cosmics are screened. But there are still fake signals, e.g., proton+nucleus interactions in the atmosphere can create pions, which decay into muons; muons penetrate deep in the rock (*Why?*) and can break nuclei in the rock, liberating protons that can enter the detector if they are produced close enough to it
- Fake signal from reaction $p + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + n$
- This background was measured at different depths with a portable C_2Cl_4 detector and extrapolated



Counting ^{37}Ar

- Large quantities of Helium were used every few months to purge the Argon gas from the liquid
- Argon is then separated from Helium by absorption in a cold charcoal trap
- Then, a radioactive counter detects $^{37}\text{Ar} \rightarrow ^{37}\text{Cl} + e^+ + \nu_e$
- How do you know the efficiency of extraction of ^{37}Ar ?
 - Control data: a known quantity of ^{36}Cl or ^{38}Cl (alternated) in the tank
 - 36 or ^{38}Ar measured by passing the gas through a mass spectrometer

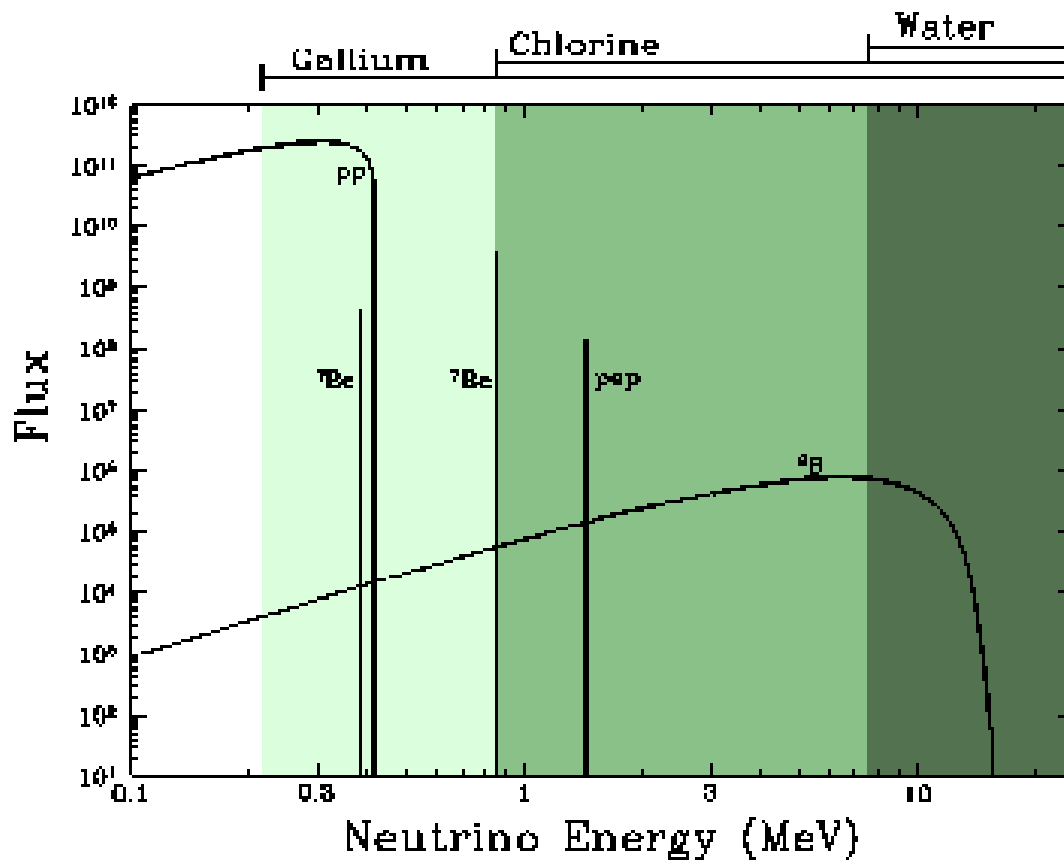


Question: why did they alternate runs with Cl^{36} and Cl^{38} ?

Solar neutrino problem

- SNU (Solar Neutrino Unit) = 10^{-36} captures/second/atom
- Homestake experiment had $\sim 2 \times 10^{30}$ atoms: a signal of 1 SNU is equivalent to one neutrino capture every 6 days (*Exercise*)
- The prediction of the Standard Solar Model (SSM) was **~ 6 SNU**
 - Background from cosmic rays was estimated as 0.25 SNU
- Homestake experiment, after 3 years of data taking: < 3 SNU
- After 30 years of data taking: **$2.56 \pm 0.16(\text{stat}) \pm 0.16(\text{syst})$ SNU**
- More precise experiments and more precise models continued to show the same disagreement for several decades
- Typical particle physicist's opinion: astrophysics is not such a precise science, SSM must be wrong
- Typical astrophysicist's opinion: SSM makes many successful predictions, it cannot be wrong

Confirmations of the problem



- GALLEX and SAGE in the '90s used $\nu_e + {}^{37}\text{Ga} \rightarrow {}^{37}\text{Ge} + e^-$ and were sensitive also to PP-I; found 60-70% of predicted flux
- Kamioka in the '80s used water ($\nu_e + e^- \rightarrow \nu_e + e^-$); found 1/2 of predicted flux
- Does the deficit depends on the process? Or on ν energy?

Solution: neutrino oscillations

- Up to that moment, no evidence that neutrinos have mass
 - (We continue having no *direct* evidence: all measurements are statistically consistent with 0)
- As you know from particle physics courses, $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ are not eigenstates of the total Hamiltonian (which includes mass terms)
- Neutrino wavefunctions: superpositions of $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$
 - Nuclear reaction or decay creates pure state $|\nu_e\rangle$
 - It can be decomposed as $|\phi(0)\rangle = |\nu_e\rangle = a|\nu_1\rangle + b|\nu_2\rangle + c|\nu_3\rangle$
 - $|\phi(t)\rangle = U(t)|\phi(0)\rangle = a'|\nu_1\rangle + b'|\nu_2\rangle + c'|\nu_3\rangle = x|\nu_e\rangle + y|\nu_\mu\rangle + z|\nu_\tau\rangle \neq |\nu_e\rangle$
 - Interaction with ^{37}Cl : only possible for $|\nu_e\rangle \Rightarrow$ acts as a **filter**
 - Observed flux is $|x|^2$ times the real ν flux

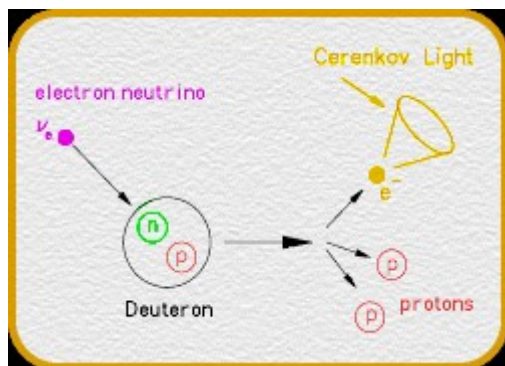
Homework #3

- Assume a simplified model with only two neutrinos
- Estimate the probability that a 1 MeV ν_e converts in a ν_μ after traveling the Sun-Earth distance
- Suggestions:
 - Consider initial state $|\phi(0)\rangle = |\nu_e\rangle$, calculate $|\langle \nu_\mu | \phi(t) \rangle|^2$
 - Expand $|\nu_{e,\mu}\rangle$ in terms of eigenstates $|\nu_{1,2}\rangle$
 - Express the mixing as function of a single parameter (θ); for example, $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$ is a valid expression
 - Expand $|\nu_\mu\rangle$ taking into account unitarity and orthogonality
 - $|\nu_{1,2}\rangle$ are eigenstates of the Hamiltonian, i.e. $f(\tilde{H})|\nu_{1,2}\rangle = f(E_{1,2})|\nu_{1,2}\rangle$
 - Use $E \gg m$ to simplify formulas

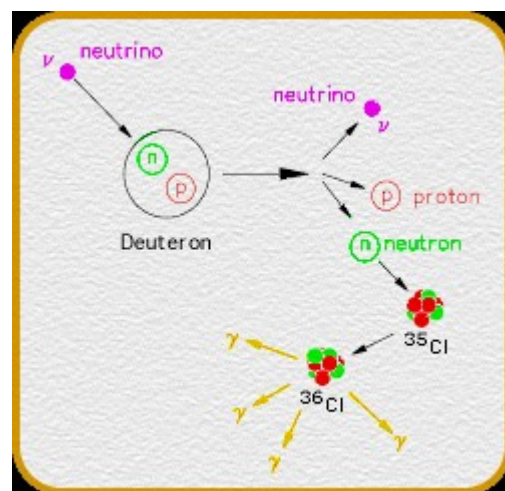
Sudbury Neutrino Observatory

- Took data in 1999-2006
- Tank filled with heavy water (D_2O), immersed in larger tank of H_2O (*)
- Runs with and without NaCl in the D_2O

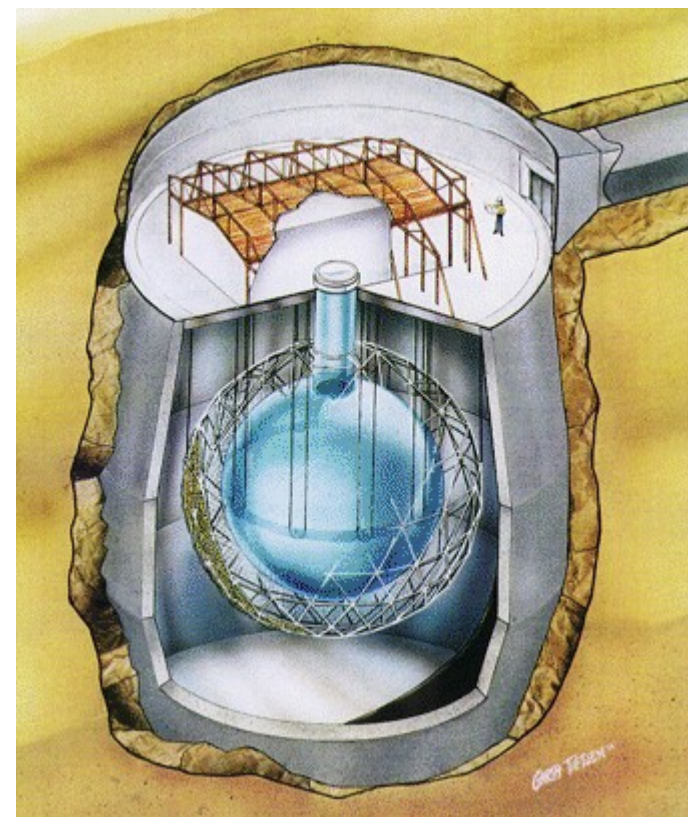
Images from [here](#)



Charged current process
(i.e., a W^+ or W^- is involved)
Only possible for ν_e
($\nu_e n \rightarrow p e^-$), blind to ν_μ and ν_τ



Neutral current process
(i.e., a Z^0 is involved)
All neutrinos contribute equally. Cl allows detection.

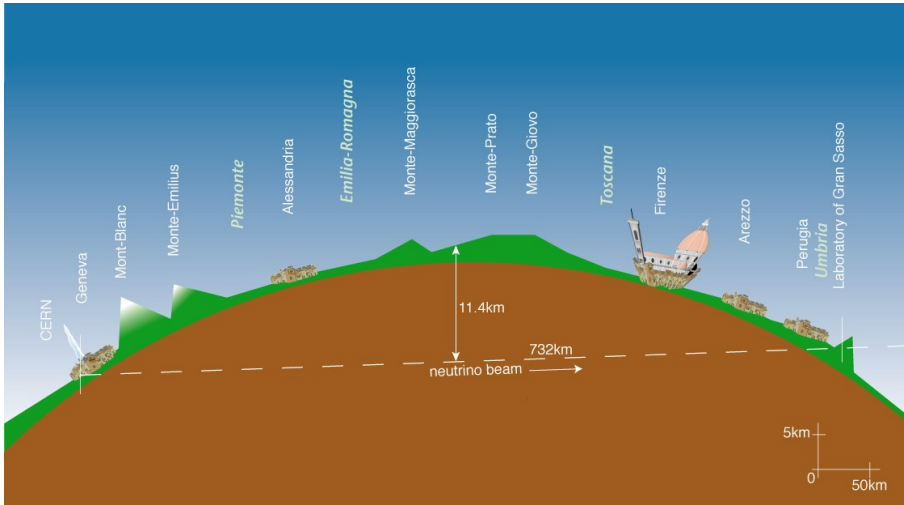


(*) The H_2O volume protected against background neutrons.

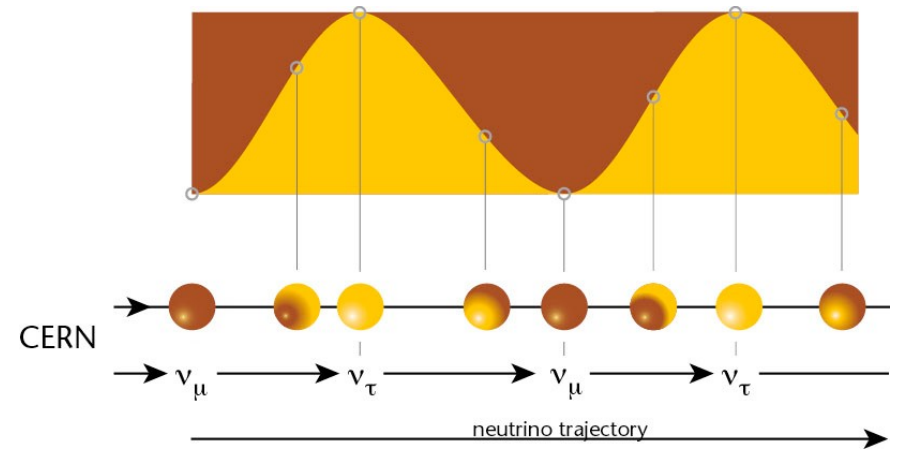
Question: what about $\nu_x + e^- \rightarrow \nu_x + e^-$?

Questions?

CERN to Gran Sasso



Images from <http://proj-cngs.web.cern.ch/proj-cngs>



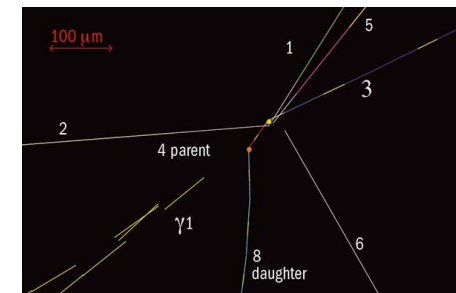
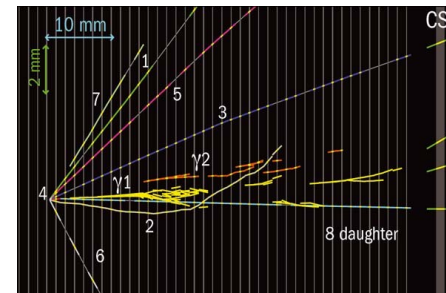
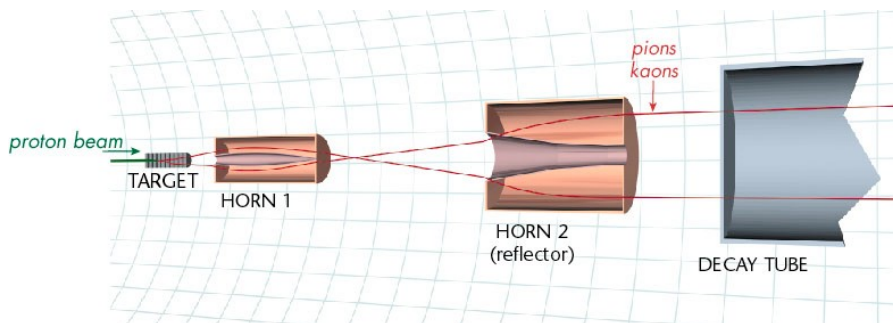
Produce a ν_μ beam at CERN:

- Smash protons against a heavy target
- Strong nuclear interaction produces many π, K
- $\pi^\pm \rightarrow \mu^\pm \nu_\mu$ (same for K^\pm)

At Gran Sasso:

- $\nu_\mu X \rightarrow \mu + Y$
- $\nu_\tau X \rightarrow \tau + Y$, followed by $\tau \rightarrow \text{hadron(s)} + \nu_\tau$

Final ν_τ deduced by momentum imbalance



Long./transv. views of an event
interpreted as $\tau^- \rightarrow \pi^- + \pi^0 (\rightarrow \gamma\gamma) + \nu_\tau$