Astrophysics and Nuclear Astrophysics

(LPHY2263)

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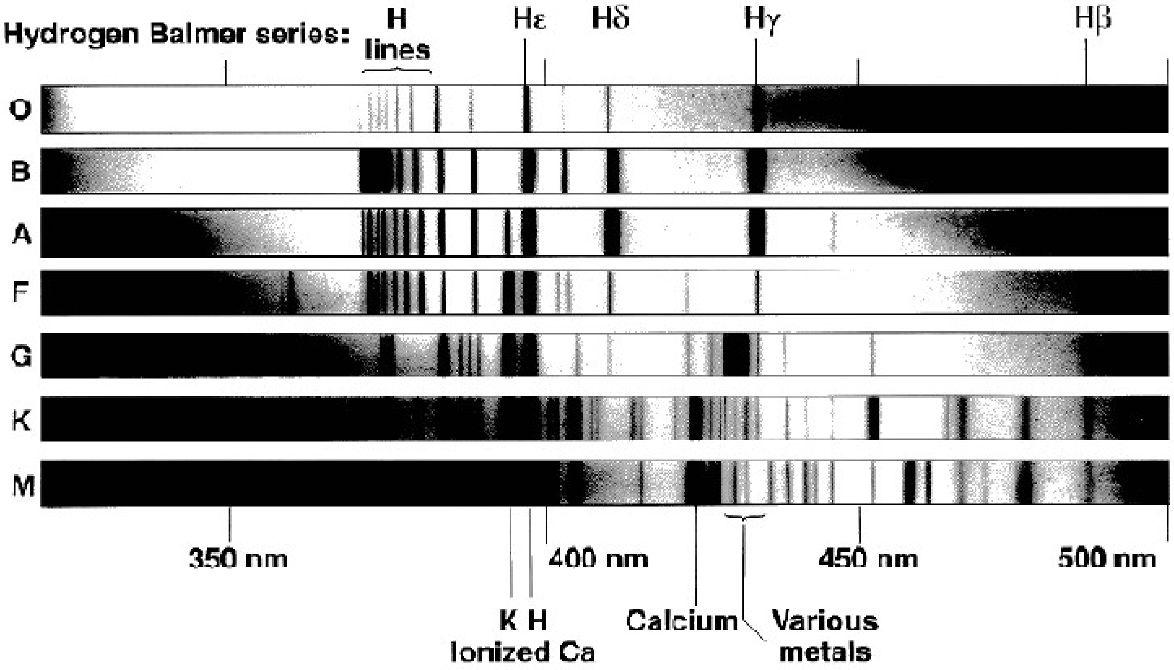
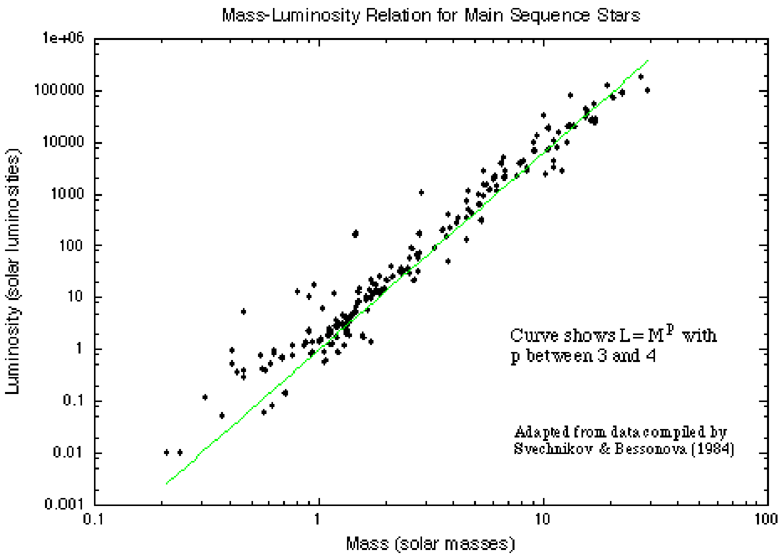
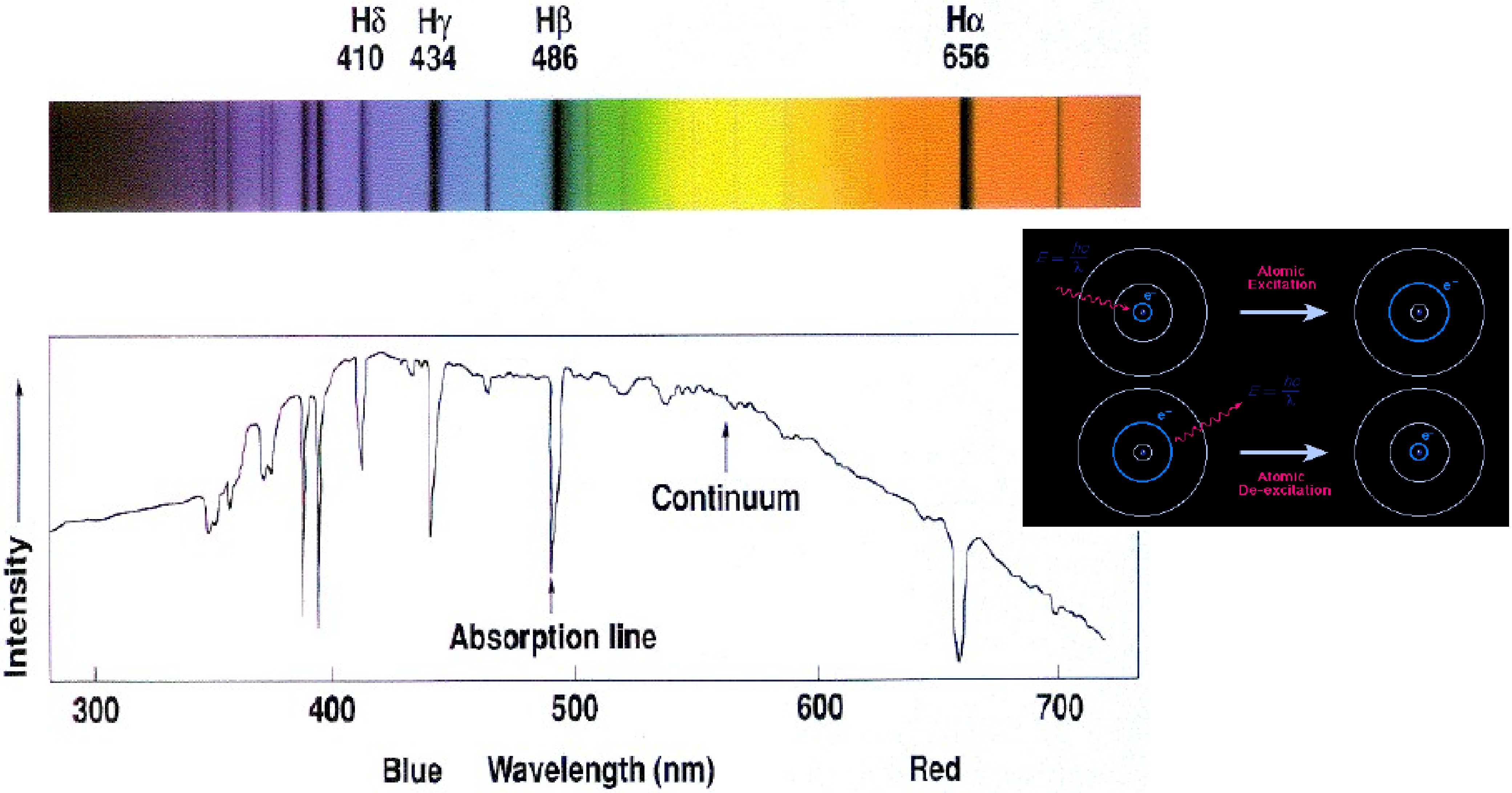
Chapter #2

- Spectrum
- Spectral type
- Luminosity versus mass



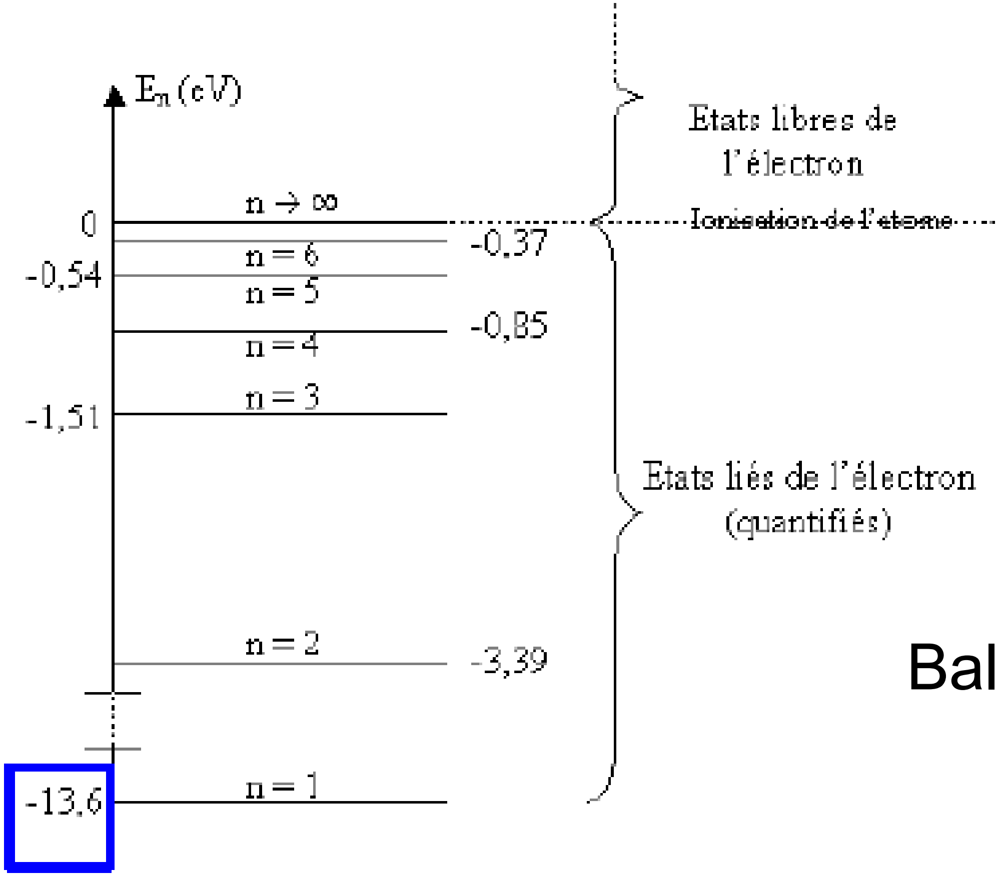


Absorption spectra

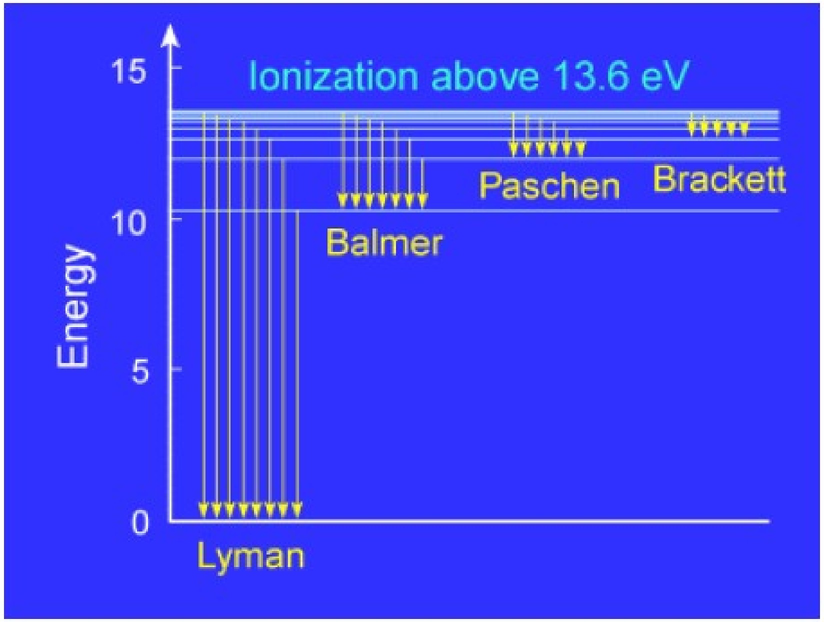


Black-body continuum spectrum filtered by passing through the photosphere

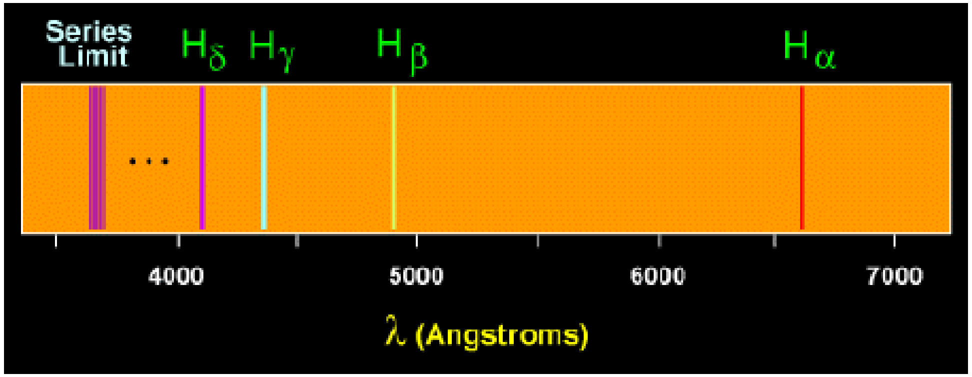
Remember the hydrogen atom



$$E_n = - 1/n^2 (me^4/2\hbar^2)$$

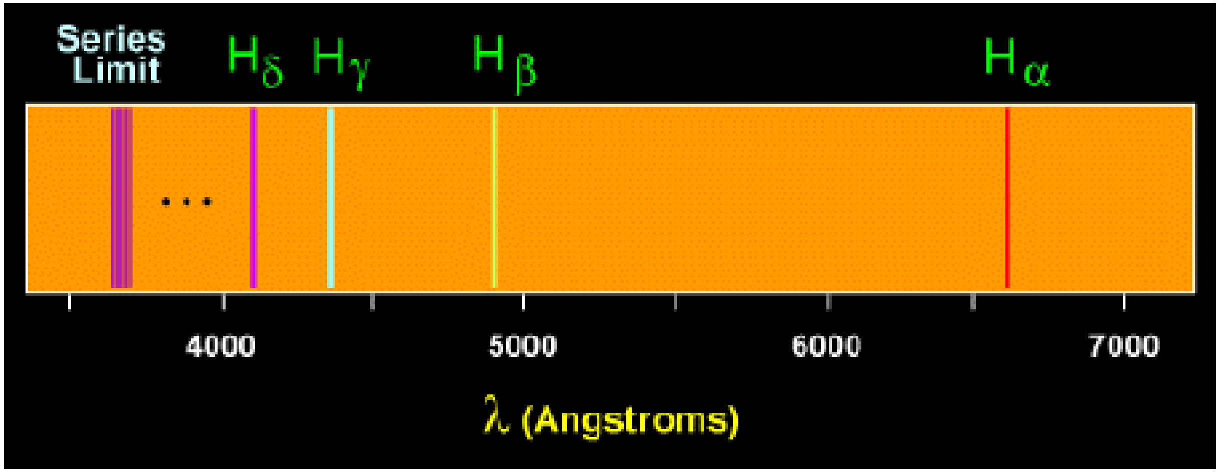


Balmer series is in the optical spectrum:



Hα : 6562 Å, Hβ : 4861 Å , Hγ : 4340 Å

Balmer series



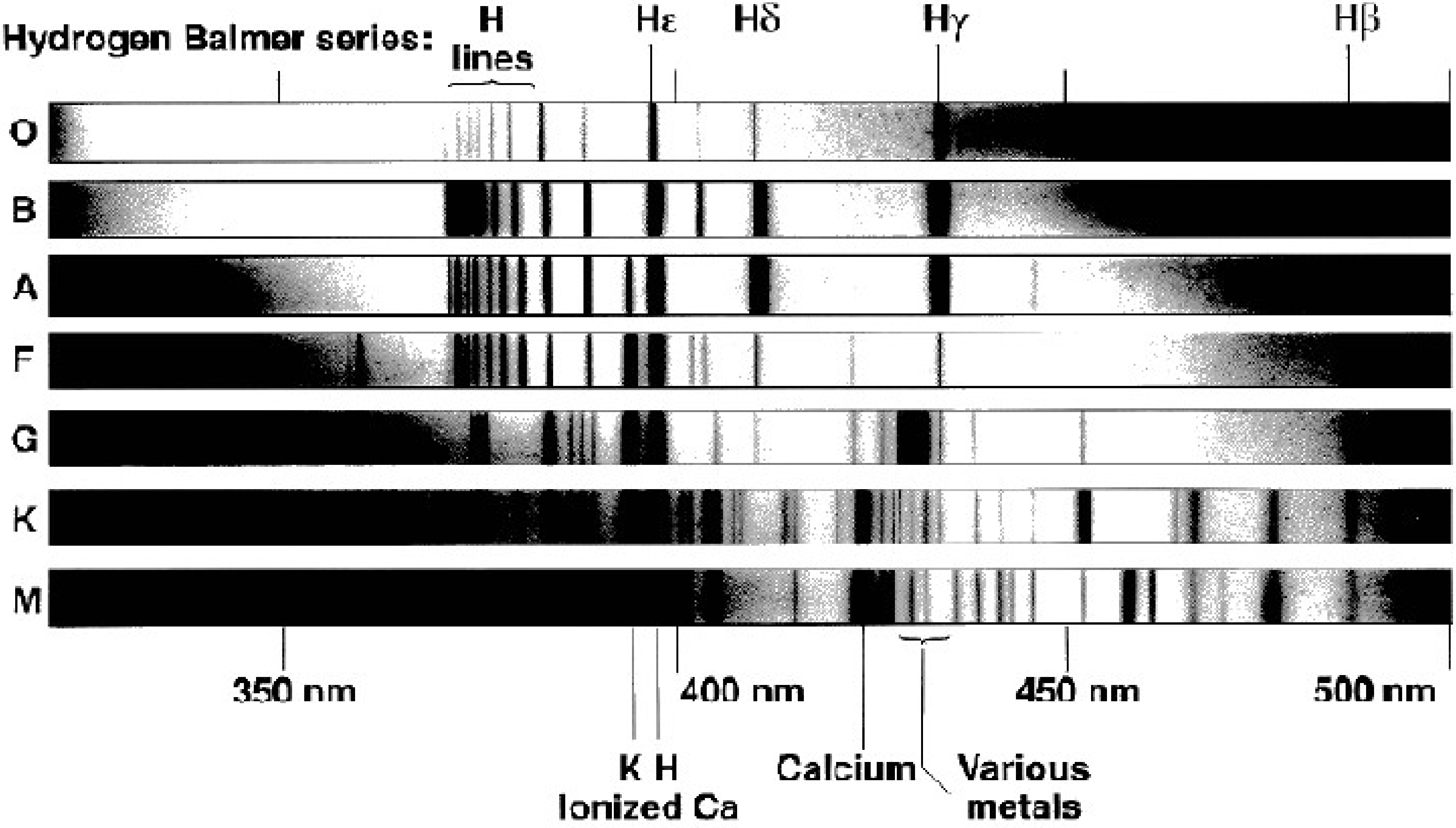
Hα : 6562 Å, Hβ : 4861 Å , Hγ : 4340 Å

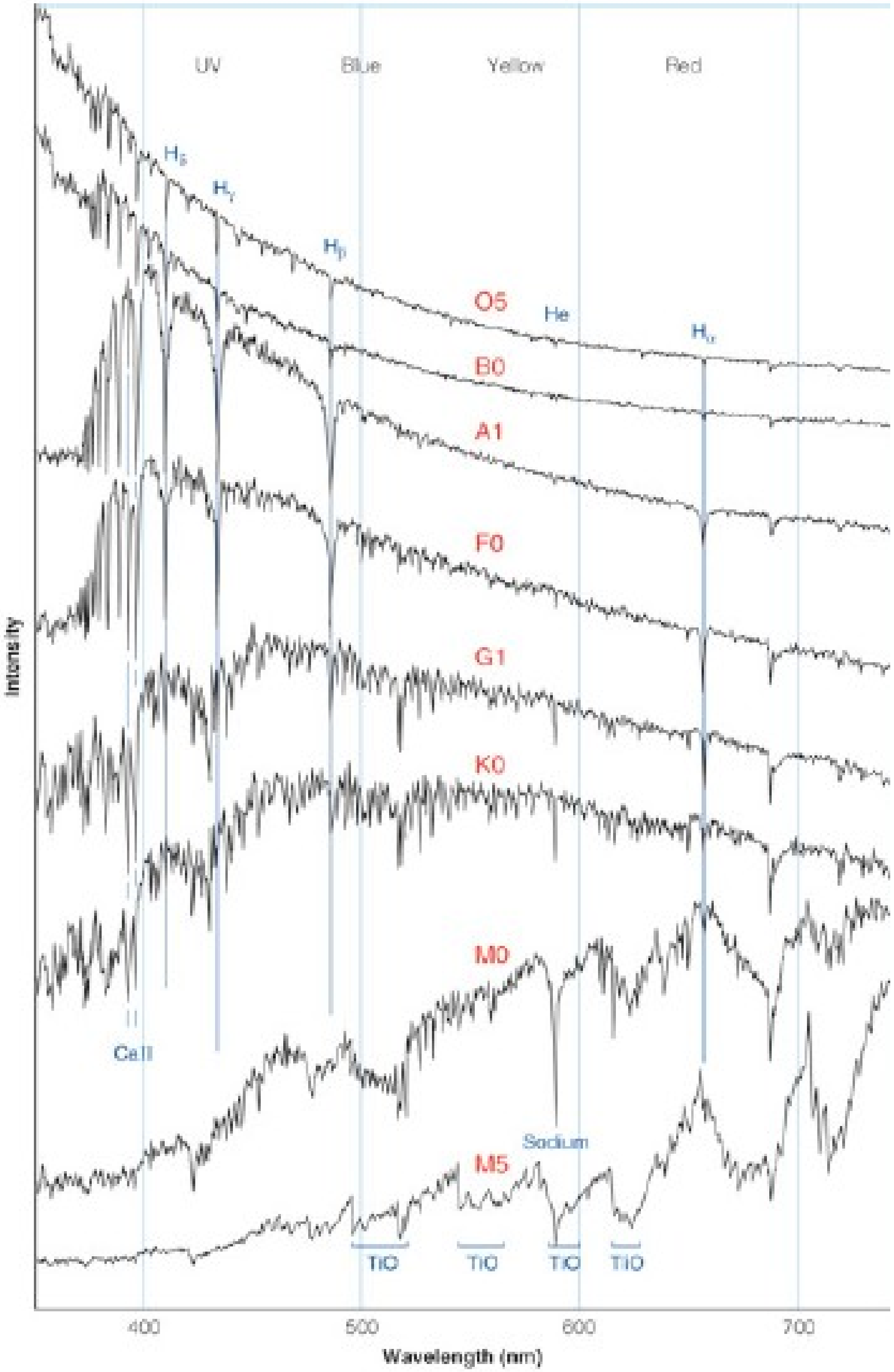
- The Balmer series is observable if:
  - There is hydrogen in the photosphere
  - There are enough hydrogen atoms in excited level n=2
- Second point depends on temperature of the photosphere
  - Remember Boltzmann Law

Classification

- First attempt by Edward Pickering:
  - Sort by hydrogen absorption-line strength
  - Type A: strongest H absorption lines, followed by B, C, ...
- Annie Jump Cannon (leader of Pickering's team of "computers") noticed a subtle pattern among metal lines
  - Rearranged Pickering's types, throwing out many as redundant
  - Order now became O, B, A, F, G, K, M
  - (Famous mnemonic trick: *Oh, Be A Fine Girl/Guy, Kiss Me*)
  - Now we know that it is an ordering in temperature
  - O: hottest and most blue; M: coldest and most red

Spectral types





Hottest

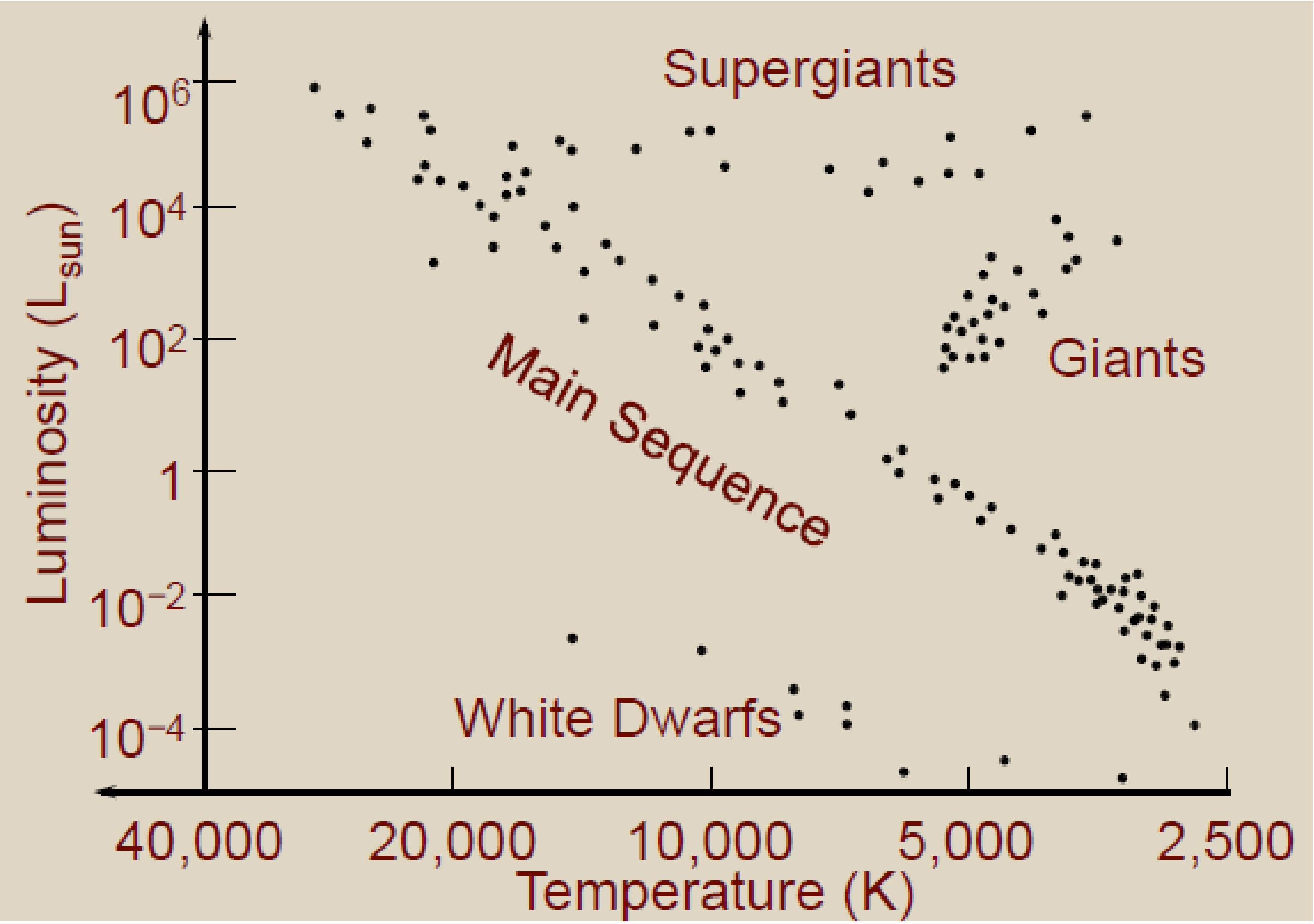
Coldest

Weak H_α

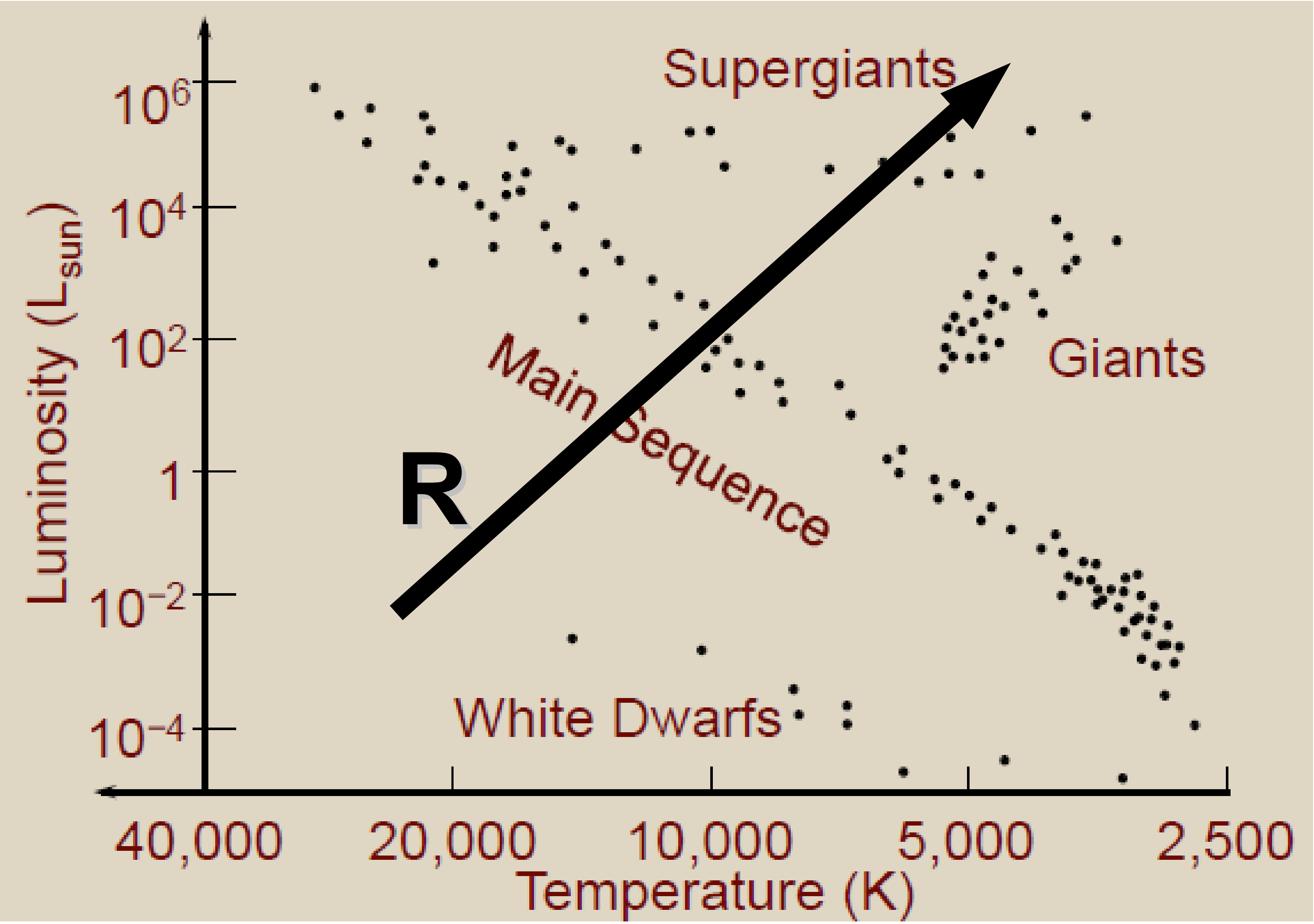
Strong H_α

Weak H_α

Hertzsprung-Russell diagram

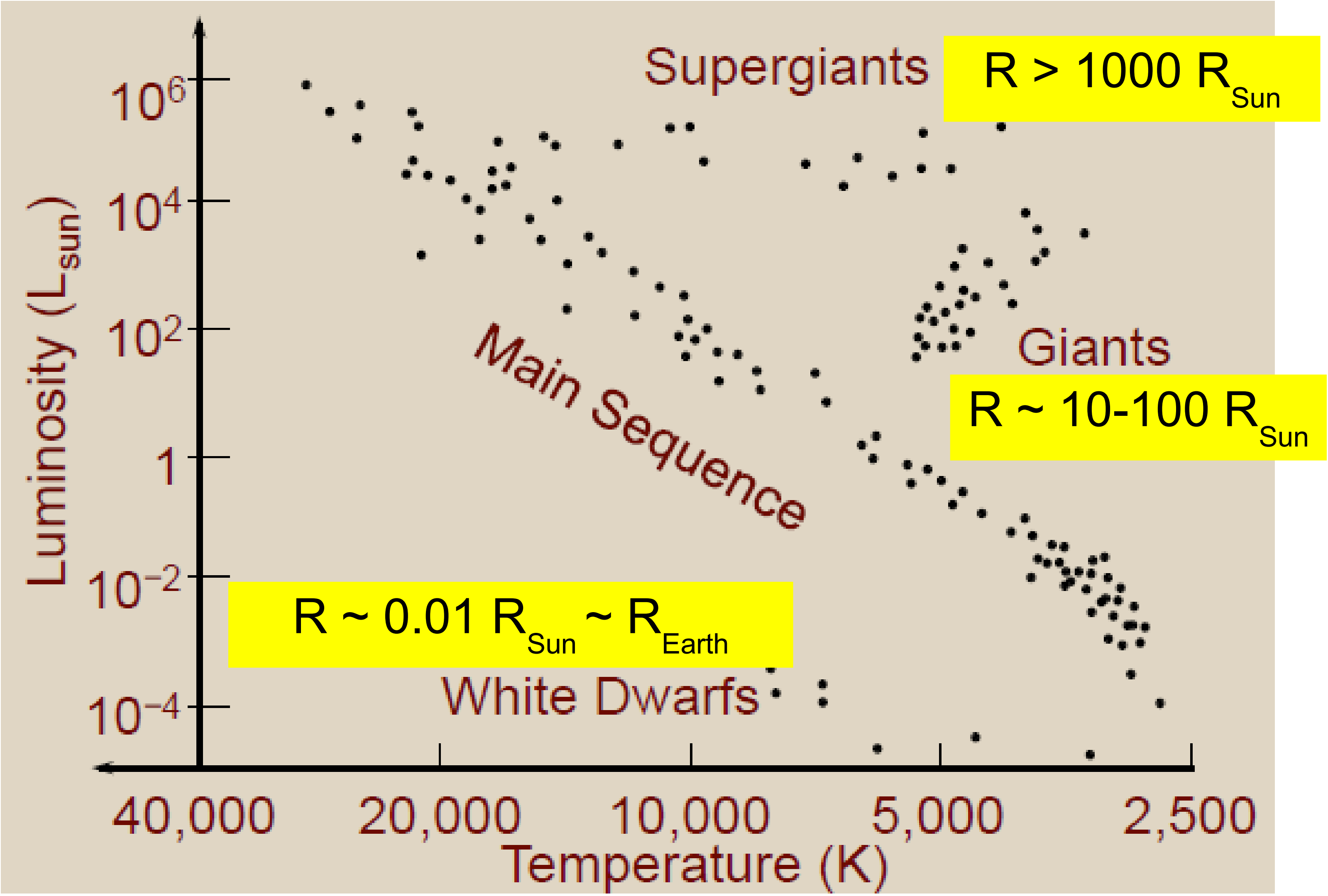


Hertzsprung-Russell diagram



Remember that size can be deduced from $L=4\pi R^2\sigma T^4$

Hertzsprung-Russell diagram



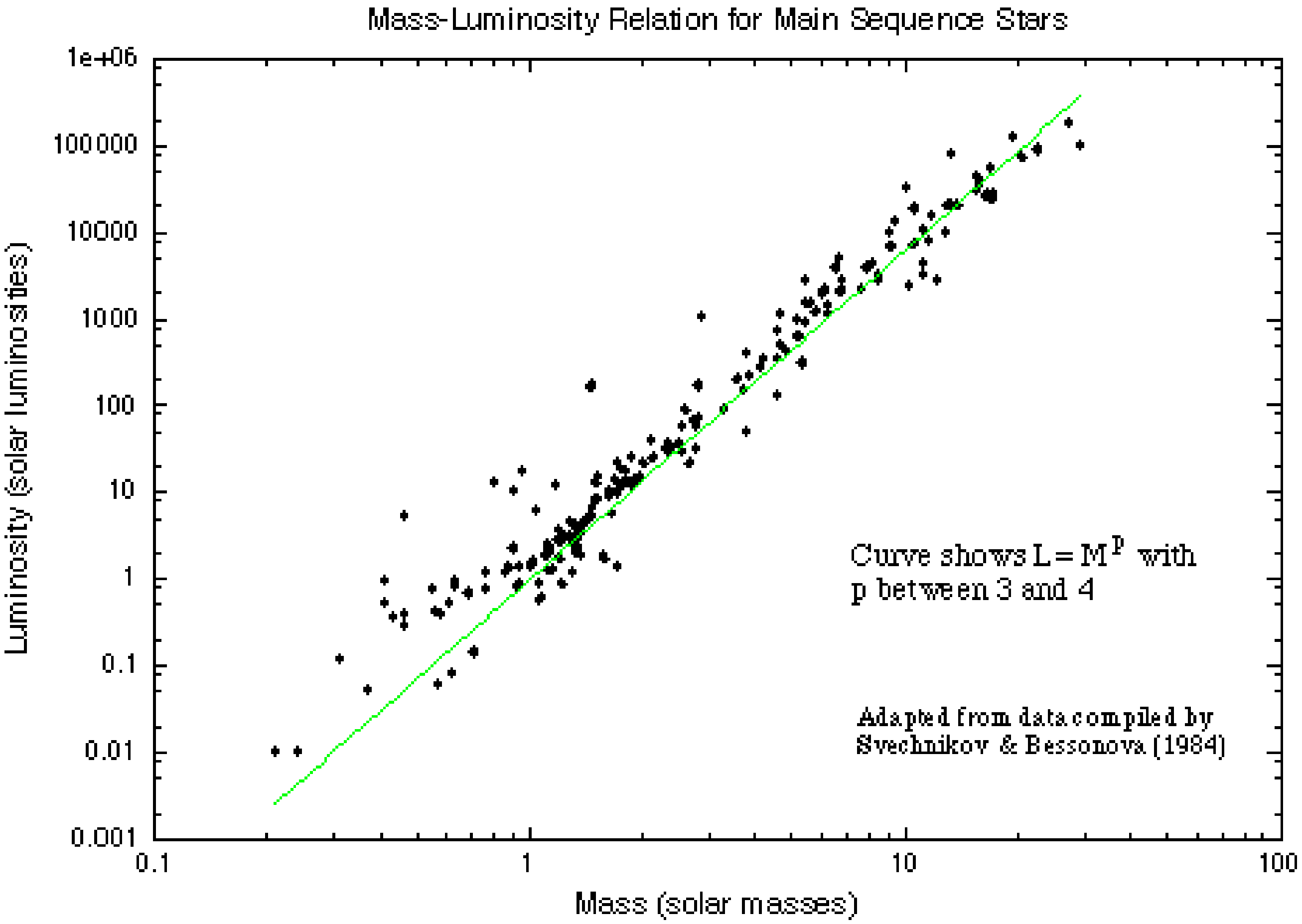
$R > 1000\ R_{Sun}$

$R \sim 10\text{-}100\ R_{Sun}$

$R \sim 0.01\ R_{Sun} \sim R_{Earth}$

85% of stars are along the Main Sequence ($R \sim 0.1\text{-}10\ R_{sun}$)

Luminosity versus mass in the Main Sequence



This image is courtesy of Nick Strobel, http://www.astronomynotes.com

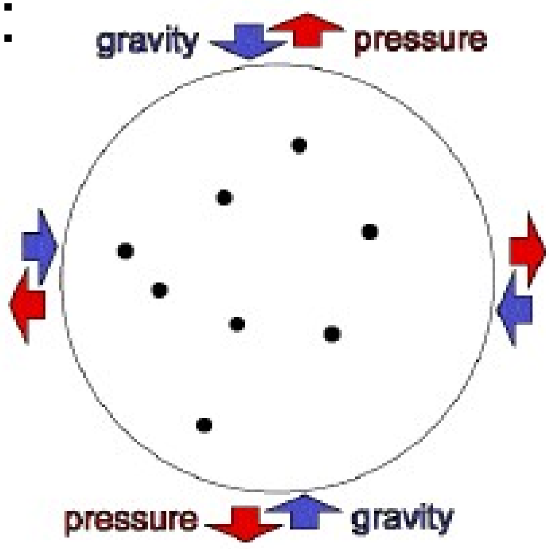
For the Main Sequences stars whose mass can be measured reliably (binaries), this **empirical** relationship is found:

$$\left(\frac{L}{L_{sun}}\right) = \left(\frac{M}{M_{sun}}\right)^{3.5}$$

In the next slides we try to explain it...

Assumption: a star is a blob of gas

- Assume a "perfect gas" kept together by gravity
- **Perfect Gas Law**:
  - Pressure is proportional to density * temperature
  - I.e., compressing the star results in higher P & T
- **Gravitational attraction** of the gas atoms:
  - Proportional to $1/R^2$
  - I.e., compressing the star results in stronger binding
- **Exact balance**: hydrostatic equilibrium

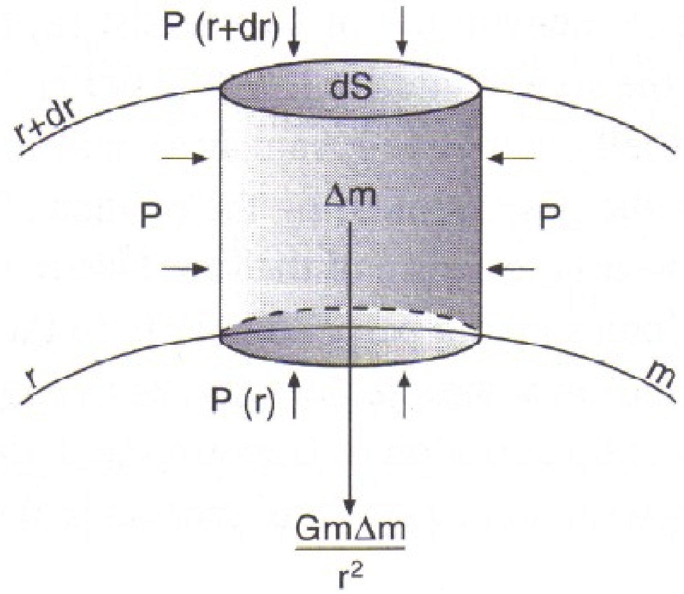


The constant fight between P and G

$$\boxed{\ddot{r}\,\Delta m = -\frac{Gm\Delta m}{r^2} + P(r)\,dS - P(r+dr)\,dS}$$

$$P(r+dr) = P(r) + (\partial P/\partial r)dr$$

$$\Delta m = \rho\,dr\,dS$$



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$$P(r+dr) = P(r) + (\partial P/\partial r)dr$$

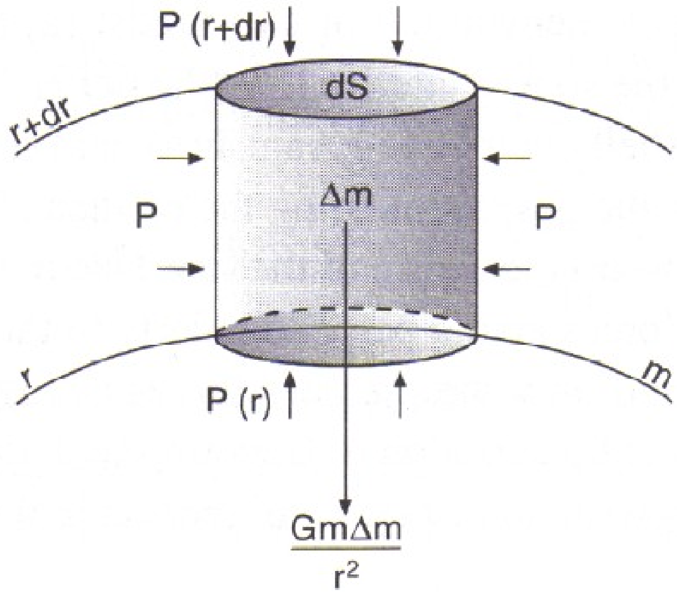
$$\Delta m = \rho\,dr\,dS$$

$$\downarrow$$

$$\ddot{r}\,\Delta m = -\frac{Gm\,\Delta m}{r^2} - \frac{\partial P}{\partial r}\frac{\Delta m}{\rho} \rightarrow \boxed{\ddot{r} = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{\partial P}{\partial r}}$$

(a)

$$\downarrow\ dr = dm/(4\pi r^2\rho)$$



The constant fight between P and G

$$\boxed{\ddot{r}\,\Delta m = -\frac{Gm\Delta m}{r^2} + P(r)\,dS - P(r+dr)\,dS}$$

$$P(r+dr) = P(r) + (\partial P/\partial r)dr$$

$$\Delta m = \rho\,dr\,dS$$

$$\ddot{r}\,\Delta m = -\frac{Gm\Delta m}{r^2} - \frac{\partial P}{\partial r}\frac{\Delta m}{\rho} \longrightarrow \boxed{\ddot{r} = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{\partial P}{\partial r}}$$

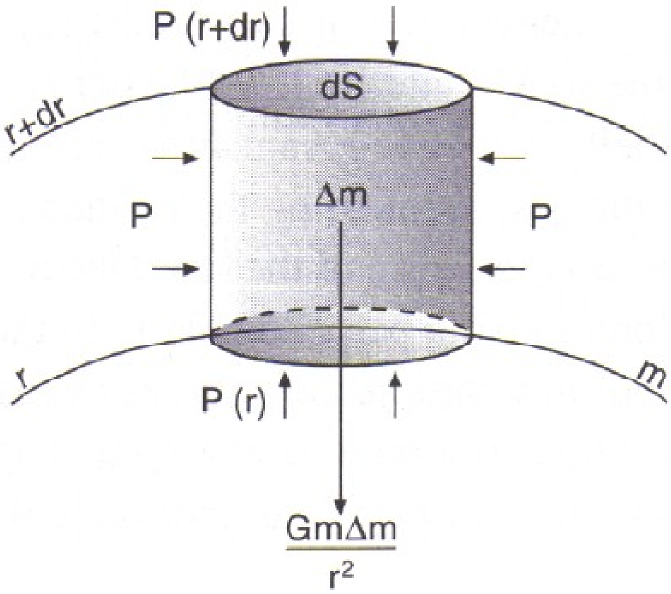
(a)

$$dr = dm/(4\pi r^2 \rho)$$

$$\ddot{r} = -\frac{Gm}{r^2} - 4\pi r^2\frac{\partial P}{\partial m}$$

$$\boxed{\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}}$$

(b)



Equilibrium means no expansion or contraction

Another way to get there

- Equilibrium means no expansion, no contraction:

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$ **(from a)**

- Integrate from 0 to R to get the average across the star:

$$\langle P \rangle = -\frac{1}{3}\frac{E_{GR}}{V}$$ **(c)** (Homework: derive this formula)

- Potential gravitational energy of a spherical mass distribution:

$$E_{GR} = -\frac{3}{5}\frac{GM^2}{R}$$ (Homework: derive this formula)

- Substitute this in equation (c) (with V=4πR^3):

$$\langle P \rangle \approx \frac{GM^2}{4\pi R^4}$$

(d); compare with (b), are they coherent?
(Homework)

The mass-luminosity relationship

- Use the ideal gas law (PV=nkT) to solve for T:

$$\langle P \rangle = \frac{\langle \rho \rangle}{\bar{m}} kT$$
$$\Rightarrow kT = \frac{GM\bar{m}}{3R}$$

- (Here $\overline{m}$ is the average mass of the gas particles in the star)
- Now use the relationship between mass and volume to get R:

$$R = \left(\frac{3}{4} \frac{1}{\rho\pi} M \right)^{\frac{1}{3}}$$

- Put all this in the eq. that we derived in the previous lesson:

$$L = 4\pi R^2 \sigma T_e^4$$

- In the end you find that L is proportional to $M^{10/3}$, i.e. $M^{3.33}$, which is very close to the observed relationship $M^{3.5}$
  - *Note: we made many approximations, not valid under all conditions*
- This can be used to infer mass from luminosity!

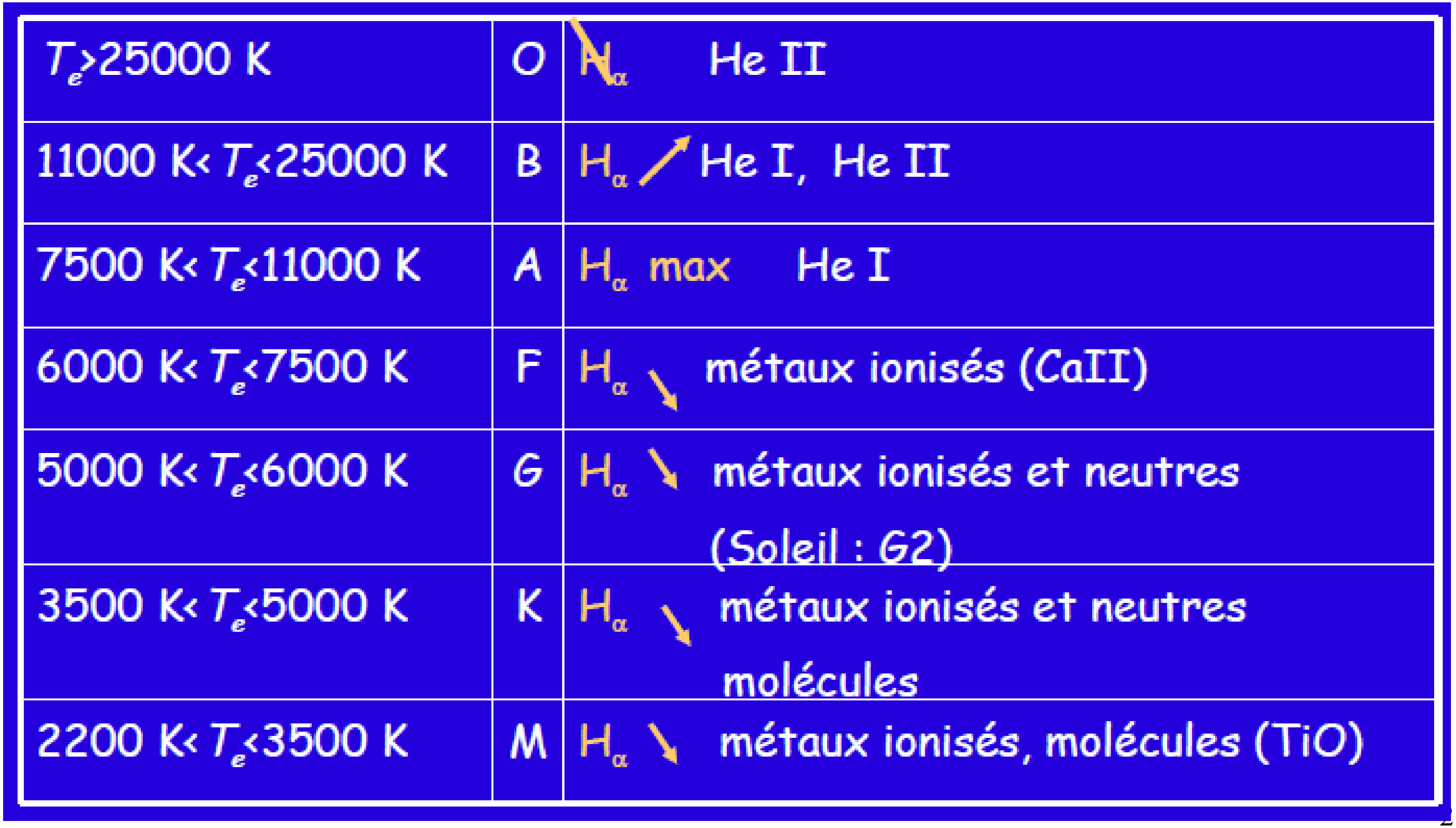
Questions?

Table of main-sequence stellar parameters[24]

Stellar Class	Radius	Mass	Luminosity	Temperature	Examples[25]
	$R/R_\odot$	$M/M_\odot$	$L/L_\odot$	K	
O6	18	40	500,000	38,000	Theta1 Orionis C
B0	7.4	18	20,000	30,000	Phi^1 Orionis
B5	3.8	6.5	800	16,400	Pi Andromedae A
A0	2.5	3.2	80	10,800	Alpha Coronae Borealis A
A5	1.7	2.1	20	8,620	Beta Pictoris
F0	1.3	1.7	6	7,240	Gamma Virginis
F5	1.2	1.3	2.5	6,540	Eta Arietis
G0	1.05	1.10	1.26	5,920	Beta Comae Berenices
G2	1.00	1.00	1.00	5,780	Sun[note 2]
G5	0.93	0.93	0.79	5,610	Alpha Mensae
K0	0.85	0.78	0.40	5,240	70 Ophiuchi A
K5	0.74	0.69	0.16	4,410	61 Cygni A[26]
M0	0.63	0.47	0.063	3,920	Gliese 185[27]
M5	0.32	0.21	0.0079	3,120	EZ Aquarii A
M8	0.13	0.10	0.0008	2,660	Van Biesbroeck's star[28]

Table from wikipedia

Spectral type, temperature and strength of Balmer's H_α line



T_e>25000 K	O	H_α ↘ He II
11000 K< T_e<25000 K	B	H_α ↗ He I, He II
7500 K< T_e<11000 K	A	H_α max He I
6000 K< T_e<7500 K	F	H_α ↘ métaux ionisés (CaII)
5000 K< T_e<6000 K	G	H_α ↘ métaux ionisés et neutres (Soleil : G2)
3500 K< T_e<5000 K	K	H_α ↘ métaux ionisés et neutres molécules
2200 K< T_e<3500 K	M	H_α ↘ métaux ionisés, molécules (TiO)

I: neutral state; II: ionized once