Astrophysics and Nuclear Astrophysics
(LPHY2263)

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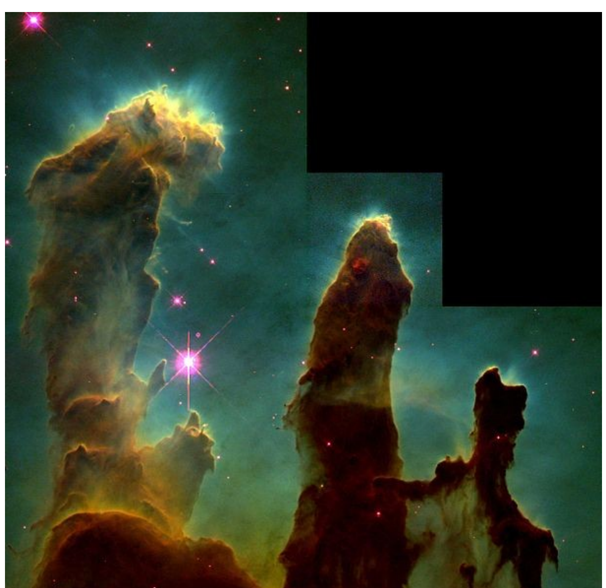
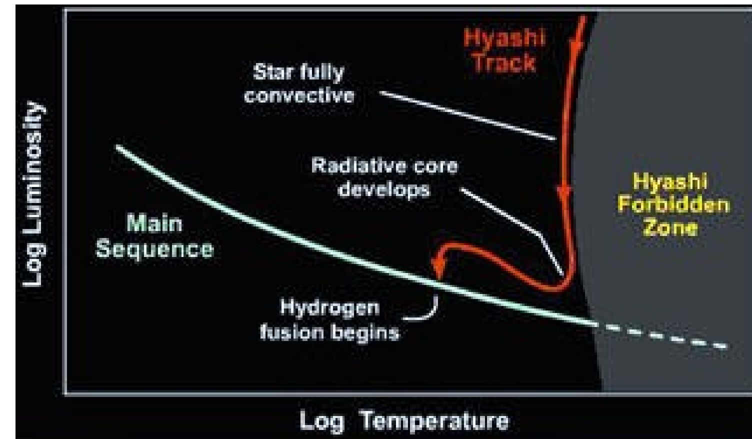
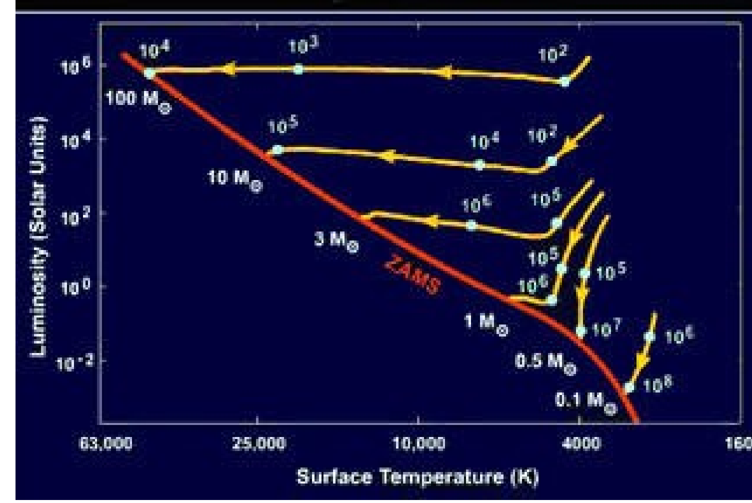
Chapter #X

- Birth of a star
  - Virial theorem
  - How contraction starts
  - Pre-main-sequence stars
  - Hayashi and Henyey tracks







Virial theorem

(demonstrations are left as exercise)

- There are several ways to formulate it
- Traditionally expressed as:

$$2\langle T\rangle = -\sum_{k=1}^{N} \langle \mathbf{F}_k \cdot \mathbf{r}_k \rangle$$

  - "virial" comes from the old name "vis viva" ("viria" is plural of "vis") for the term F•r
- The most useful equivalent form is probably:

$$2\langle T\rangle = n\langle V_{\mathrm{TOT}}\rangle$$

  - This is valid when the potential has spherical symmetry (↔ only depends on distance) and can be expressed in the form $V(r) \sim r^n$
  - In the case of gravitation, n = -1

Main forces acting on a gas cloud

- Gravitational potential energy
  - It tends to make the gas contract
- Internal thermal energy
  - It tends to make the gas expand
- Rotational energy
  - Creates a centrifugal force
- Magnetic energy
  - Molecules are polarised (in random directions), and the galaxy has a magnetic field

Internal thermal energy

- Virial Theorem ⇒ contraction can happen only if $\overline{E}_{kin} < \frac{1}{2}\overline{E}_{gr}$
- Use relationship between $\overline{E}_{kin}$ and temperature:

$$2\left(\frac{3\rho kT}{2\mu m_h}\right)\left(\frac{4\pi R_c^3}{3}\right) \leq \frac{GM^2}{R_c}$$

$$\Rightarrow R_c \leq \frac{GM\mu m_h}{3kT} \approx \frac{0.25(M/M_\odot)}{T}$$

(Jeans length)

R in parsec, T in Kelvin

- Below "Jeans length", cloud is unstable by gravity
- Typical interstellar temperature in our galaxy: ~50 K
  - So, to form the Sun, our cloud should have had a radius of less than 5×10^{-3} pc ⇒ density ~ 10^8 atoms/cm^3
  - But this is orders of magnitude more than typical clouds!

Rotational energy

- Now consider that our cloud is part of a galaxy, and this galaxy is rotating, so there is a velocity gradient between the extremes of the cloud ⇒ the cloud spins too
- Use Virial Theorem again, and assume *uniform* sphere:

$$2\left(\tfrac{1}{2}I\omega^2\right) \leq \frac{GM^2}{R_c}$$

(A little help here)

$$\Rightarrow \quad R_c \leq \left(\frac{5GM}{2\omega^2}\right)^{1/3}$$

- From the known rotation speed our our galaxy:

$$R_c \leq 0.9\left(\frac{M}{M_\odot}\right)^{1/3}$$

R in parsec

- This means that, in order for gravity to overcome rotation, the Sun's mass should have been all within 0.7 pc

Magnetic energy

- Molecules have a small magnetic momentum
- Interstellar galactic space has magnetic field of ~ 5 μG
- Again using the Virial Theorem (*Homework*):

$$R_c \leq 0.17 \left(\frac{M}{M_\odot} \right)^{1/2}$$

R in parsec

- In order for gravity to overcome magnetic repulsion, the Sun's mass should have been all within 0.13 pc
- But now let's consider rotation and magnetism together:
  - Rotating system with magnetic field ⇒ large EM effects
  - Stellar wind is accelerated outward, taking away angular momentum from the system ⇒ rotation decreases ⇒ smaller magnetic energy in the system, too

Solution to the problem

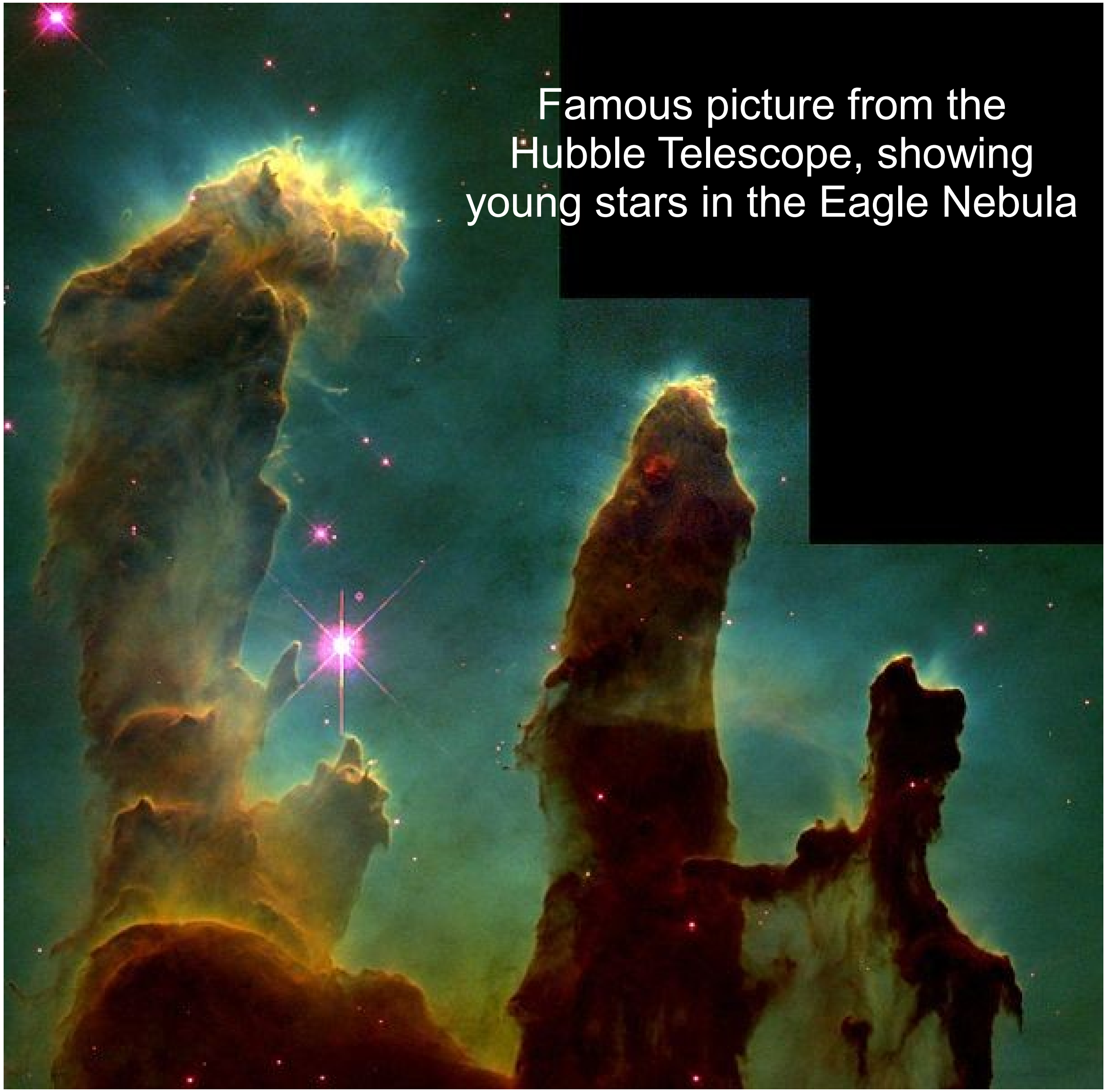
- So, rotational and magnetic effects are damped spontaneously by their interplay, but what about thermal energy?

$$R_c \leq \frac{GM\mu m_h}{3kT} \approx \frac{0.25(M/M_\odot)}{T}$$

R in parsec

- Remember that $M=\rho V \sim \rho R^3$, so one gets $R_c \sim \sqrt{T/\rho}$: for a given temperature, Jeans length increases with decreasing density, while Jeans mass increases as the cube of Jeans length
- A cloud of typical density collapses if it is of order $\sim 10^4 M_{sun}$
- The collapse of these large clouds is not completely smooth because of small inhomogeneities
- The pressure from the large contraction creates the correct local density for the formation of small stars
- Indeed stars of the same age tend to appear in clusters

"Pillars of Creation"

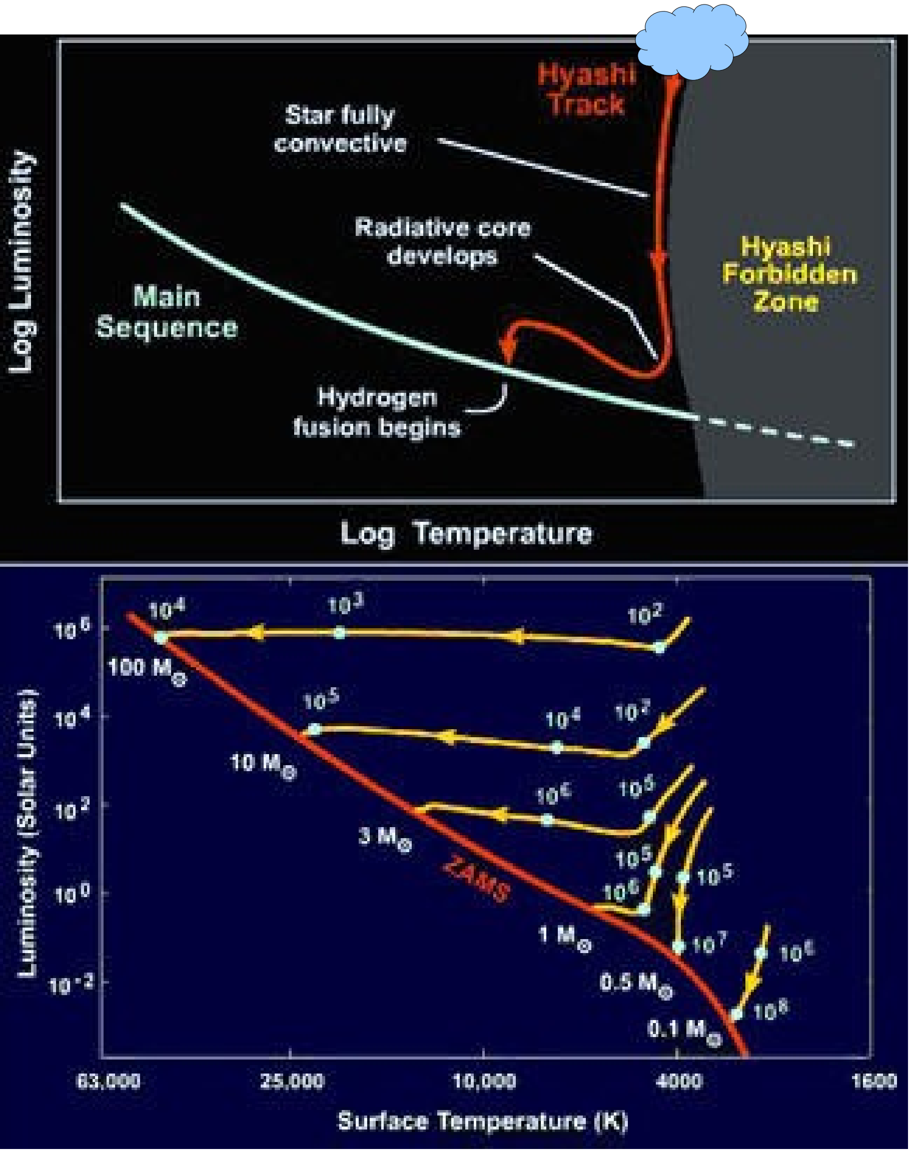


From wikipedia

Protostellar cloud

- The original giant cloud tends to fragment in smaller clouds, that contain the mass of the future star
- Collapse releases gravitational energy, causing heating
- The protostellar cloud shines by the mechanism that was studied by Kelvin and Helmoltz
  - See Chapter 3 of these lessons
  - By doing the exercise proposed there, you will know how long was this collapse phase for the Sun
- At this point the cloud gas is very luminous, $L=O(10)L_{sun}$, T~8000 K
- Over the next 1 million years, the protostellar cloud slowly contracts and cools to around 4500K

Pre-main-sequence star (1)



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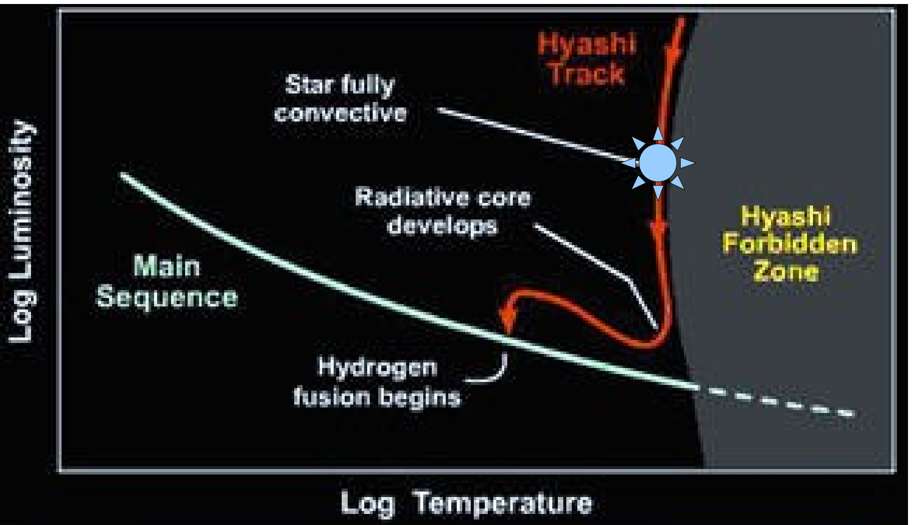
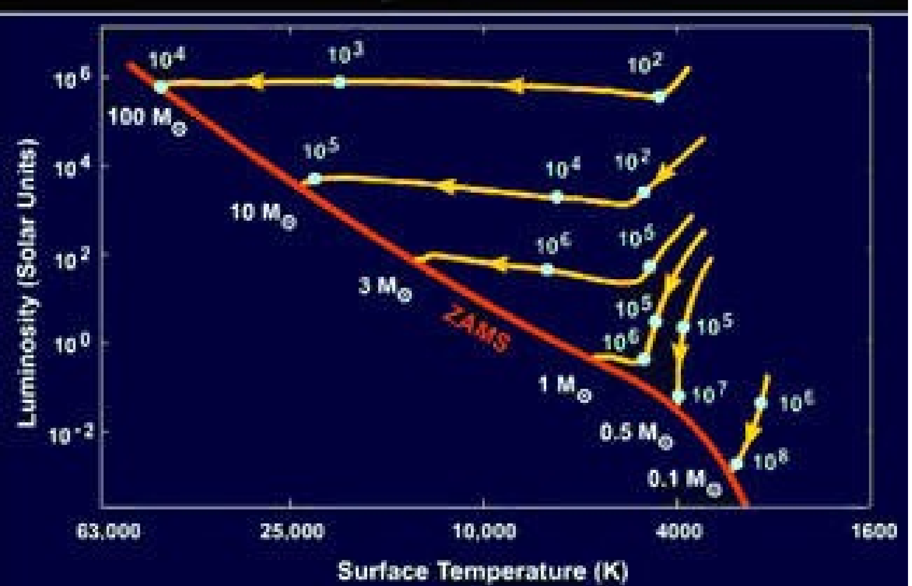
- While the surrounding matter in the cloud is falling onto the centre, it is called a **protostar**
- When the surrounding gas/dust envelope disperses and accretion process stops, the star is called **pre-main-sequence star (PMS)**
- The point in the H-R diagram where it becomes optically visible is called **stellar birthline**
- Pre-main-sequence stage lasts O(1%) of a star's life
    - Sun: ~10 million years

Pre-main-sequence star (2)





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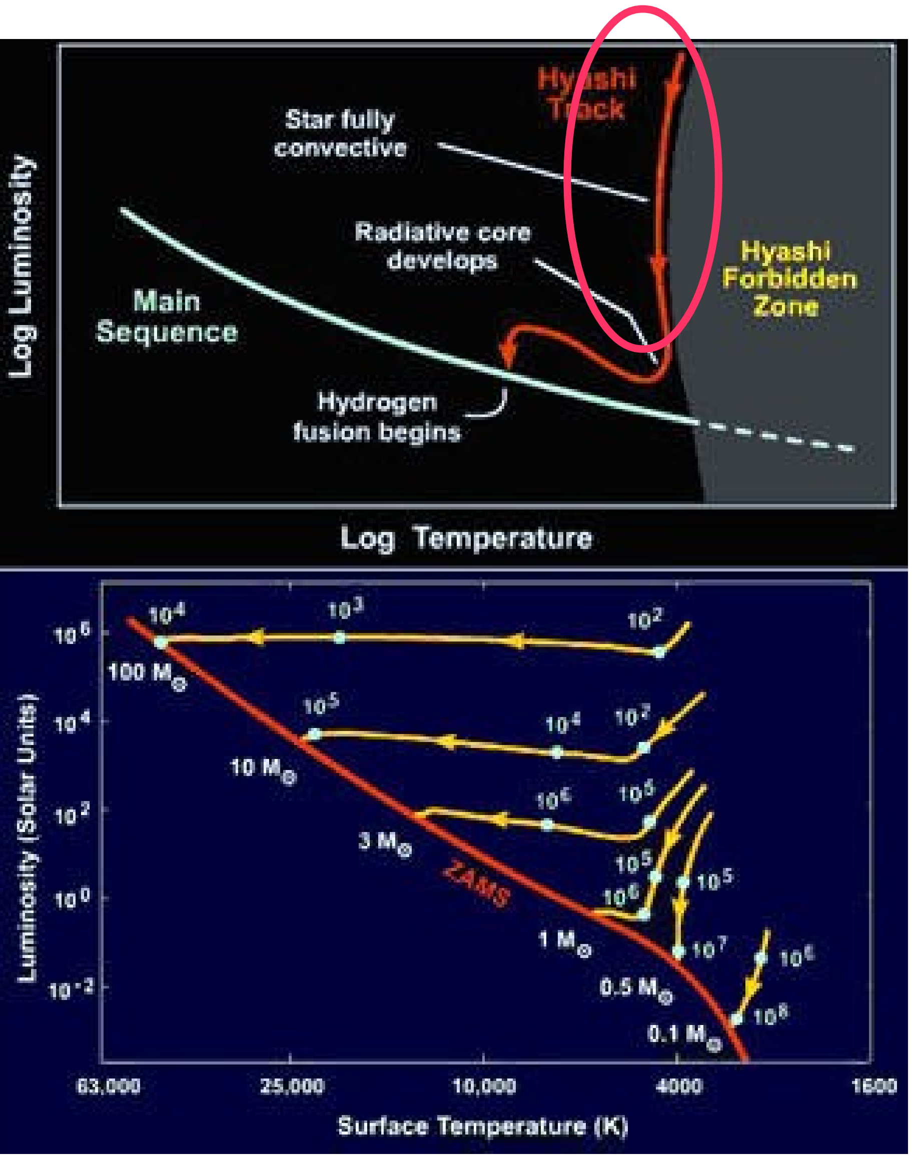
- PMS stage of stars with masses more than $0.5M_{sun}$ moves along Hayashi tracks (almost vertically down) and later along Henyey tracks (almost horizontally to the left, towards the main sequence)
- PMS stage of stars with masses less than $0.5M_{sun}$ moves along the Hayashi track for the entirety of their pre-main-sequence stage
- PMS stage ends with start of H fusion (ZAMS: zero age main sequence)

Hayashi track (1)



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- Temperature changes little, but luminosity decreases, because radius decreases (some math in next slide)
- Main property is that the star is fully convective at this stage
  - Because center is opaque to radiation; see Chapter 4 of these lessons
- Hayashi track ends when T in the center is high enough to decrease opacity down to the level that a radiative zone develops
- Below $0.5M_{sun}$ it never happens

Hayashi track (2)

- Virial theorem implies that half of the change in gravitational energy contributes to luminosity
- Luminosity is the rate of energy emission vs time, so:

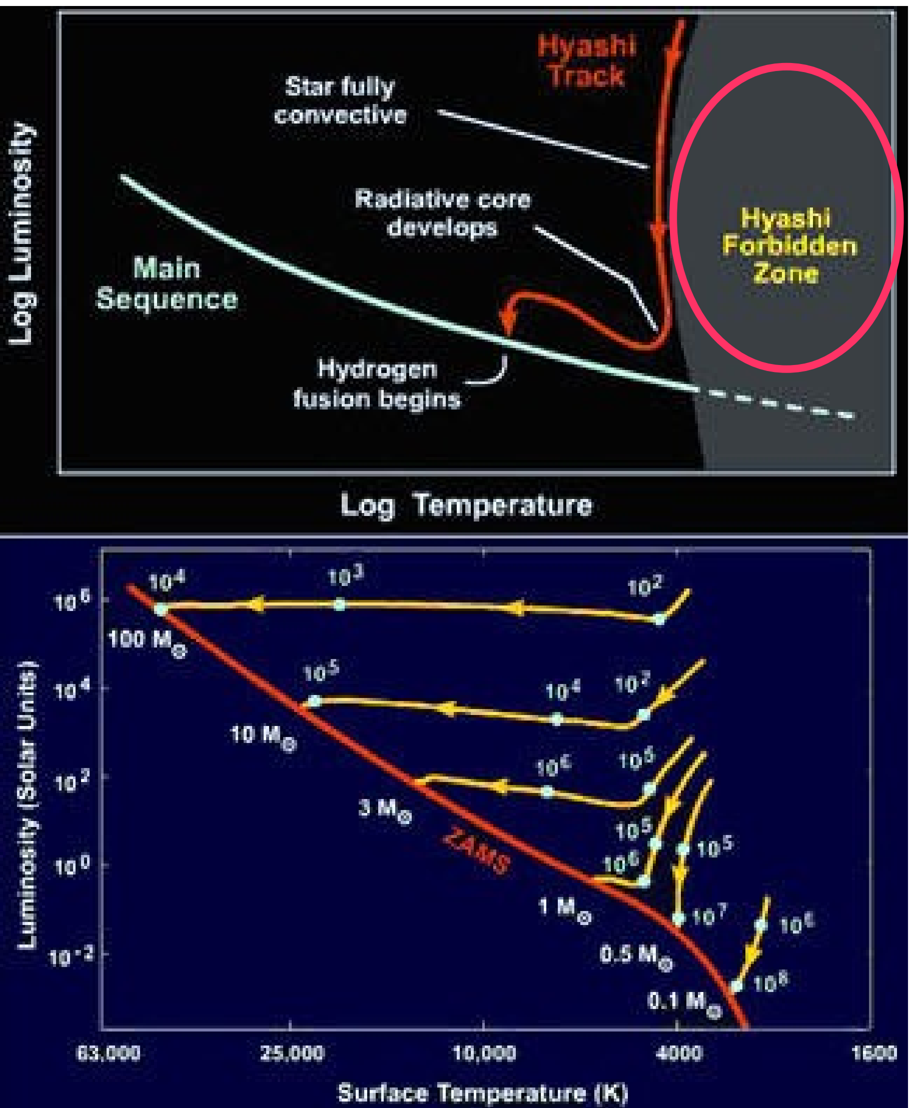
$$L = \frac{1}{2}\frac{d(GM^2/R)}{dt} = -\frac{\frac{1}{2}GM^2}{R^2}\frac{dR}{dt}$$

- L is positive ⇒ dR/dt is negative ⇒ the star contracts

$$L = 4\pi R_*^2 \sigma T_e^4 \quad \Rightarrow \quad \frac{dL}{dR_*} = \cancel{\frac{4L}{T_e}\frac{dT_e}{dR_*}} + \frac{2L}{R_*}$$

- Convection brings energy very efficiently to the surface, where it is radiated away ⇒ dT/dR ~ 0
  - *(Question: explain this last logical passage)*

Hayashi forbidden zone



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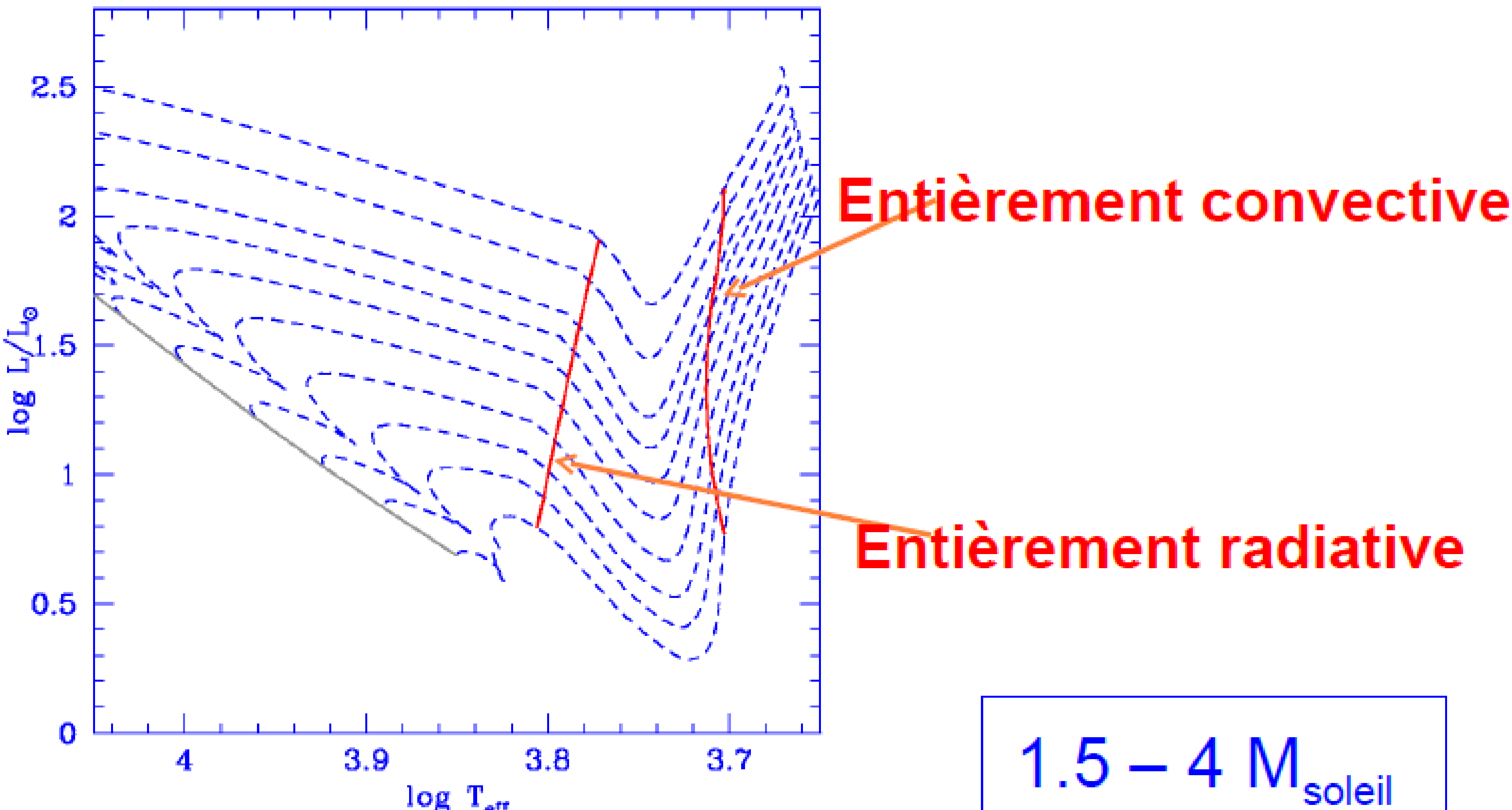
- For each mass there is a "forbidden zone", right of the Hayashi track
- No mass of given M is ever there, with the exception of protostars, which are initially there and then quickly reach the Hayashi track
- This zone is where there is no hydrostatic equilibrium
- On the Hayashi track, star radiates like a black body and is in convective equilibrium ⇒ no other configuration can lose energy more efficiently ⇒ no star on its right

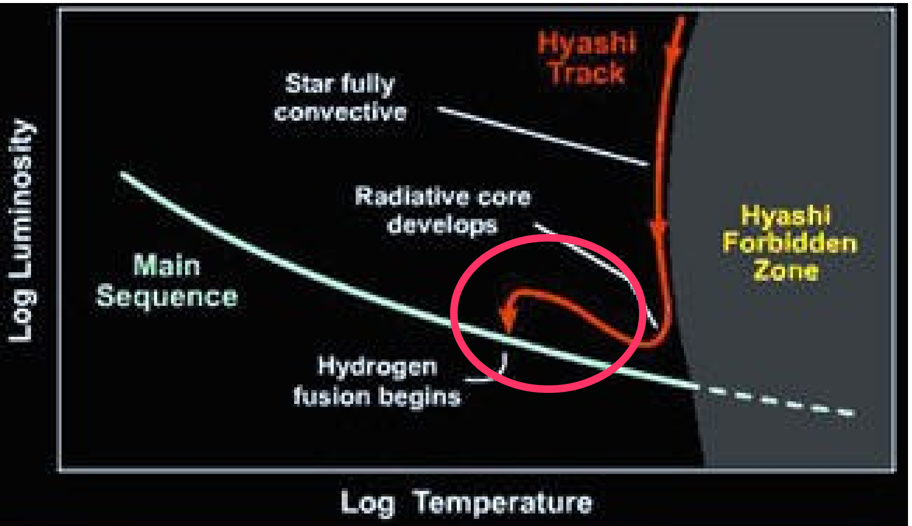
From Hayashi to Henyey

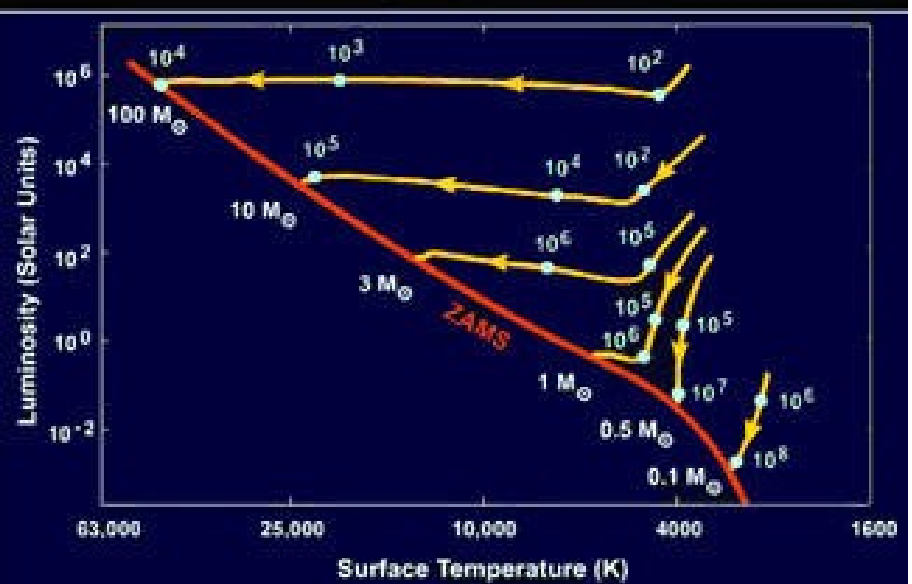


1.5 – 4 M_{soleil}

When sufficiently low opacity is reached,
convection stops and transfer becomes radiative

Henyey track





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- Radiative core expands until all star is in radiative equilibrium, and Henyey track starts
- Slow collapse ($\Rightarrow$ becomes hotter) in near hydrostatic equilibrium
- Radiative flux increases $\Rightarrow$ L grows a bit, despite decreasing radius
- It ends when temperature reaches the threshold for H fusion
- (Unless $M<0.08M_{sun}$; in that case, star is a "brown dwarf")

Time spent in pre-main-sequence

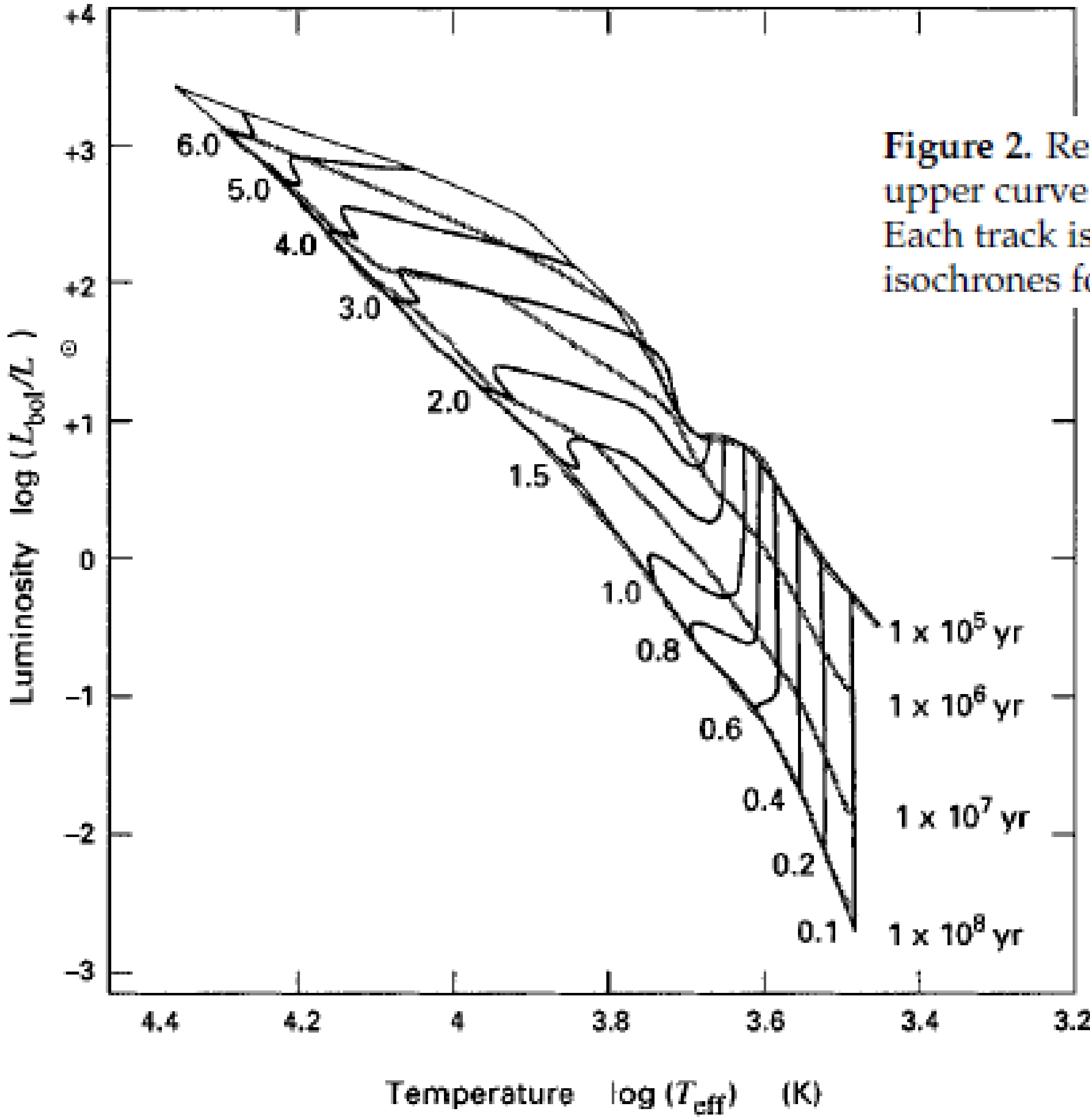

Figure 2. Recent calculation of pre-main-sequence tracks. The upper curve is the stellar birthline, and the lower one the ZAMS. Each track is labeled by the mass in solar units. Also shown are isochrones for selected times.

Taken from the *Encyclopedia of Astronomy and Astrophysics* – Nature Publishing

L-T evolution inside the Main Sequence

- When a star enters the Main Sequence (ZAMS), its H content is much higher than when it is older
- Fusion increases the He fraction, hence the mean molecular weight of the star
- To maintain the pressure, density increases by contraction of the core
- $\Rightarrow$ T increases in the core $\Rightarrow$ nuclear fusion rate increases $\Rightarrow$ L increases
- Sun is 30% more luminous today than 5 billion years ago, when it entered the Main Sequence

A digression on climatology

- This steady change in Sun luminosity versus time would have been expected to have a visible effect on Earth's climate
- But we have strong indications that Earth surface temperature changed much less than expected from this effect
- This is known as the "faint early Sun problem"
- Today we believe that the stability of our climate across billions of years is due to strong feedback mechanisms in the atmosphere